the force required by the man, power to raise the casting and the power supplied by drum run by a prime-mover. Take $\mu = 0.3$.

(226 N; 3.698 kW; 3.613 kW)

26. A leather belt transmits 10 kW from a motor running at 600 rpm by an open-belt drive. The diameter of the driving pulley of the motor is 350 mm, centre distance between the pulleys is 4 m and speed of the driven pulley is 180 rpm. The belt weighs 1100 kg/m^3 and the maximum allowable tension in the belt is 2.5 N/mm^2. $\mu = 0.25$. Find the width of the belt assuming the thickness to be 10 mm. Neglect the belt thickness to calculate the velocities.

(73.8 mm)

27. Two pulleys mounted on two parallel shafts that are 2 m apart are connected by a crossed belt drive. The diameters of the two pulleys are 500 mm and 240 mm. Find the length of the belt and the angle of contact between the belt and each pulley. Also, find the power transmitted if the larger pulley rotates at 180 rpm and the maximum permissible tension in the belt is 900 N. The coefficient of friction between the belt and pulley is 0.28.

(5.23 m, 201.4°, 2.658 kW)

28. Determine the maximum power that can be transmitted through a flat belt having the following data:
X-section of the belt = 300 mm × 12 mm
Ratio of friction tensions = 2.2
Maximum permissible tension in belt = 2 N/mm^2
Mass density of the belt material = 0.0011 g/mm^3

(64.46 kW)

29. A V-belt weighting 1.6 kg/m run has an area of cross-section of 750 mm^2. The angle of lap is 165° on the smaller pulley which has a groove angle of 40°. $\mu = 0.12$. The maximum safe stress in the belt is 9.5 N/mm^2. What is the power that can be transmitted by the belt at a speed of 20 m/s?

(82.485 kW)

30. A leather belt transmits 8 kW of power from a pulley that is 1.1 m in diameter running at 200 rpm. The angle of lap is 160° and the coefficient of friction between belt and pulley is 0.25. The maximum safe working stress in the belt is 2.2 N/mm^2. The thickness of the belt is 8 mm and the density of leather is 0.001 g/mm^3. Find the width of the belt taking centrifugal tension into account.

(78.6 mm)

31. A rope drive transmits 40 kW at 120 rpm by using 15 ropes. The angle of lap on the smaller pulley which is 300 mm in diameter is 165°. Coefficient of friction is 0.25 and the angle of groove is 40°. The rope weighs $(50 \times 10^{-6})\, G^2$ kg per metre length of rope and the working tension is limited to $0.14\, G^2$ N where G is the girth (circumference) of rope in mm. Determine the initial tension and the diameter of each rope.

(903.8 N; 34.2 mm)

32. The smaller pulley of a flat belt drive has a radius of 220 mm and rotates at 480 rpm. The angle of lap is 155°. The initial tension in the belt is 1.8 kN and the coefficient of friction between the belt and the pulley is 0.3. Determine the power transmitted by the belt.

(15.3 kW)

33. A rope drive uses ropes weighing 1.6 kg/m length. The diameter of the pulley is 3.2 m and has 12 grooves of 40° angle. The coefficient of friction between the ropes and the groove sides is 0.3 and the angle of contact is 165°. The permissible tension in the ropes is 870 N. Determine the speed of the pulley and the power transmitted.

(80.3 rpm, 86.18 kW)

34. A man wants to lower an engine weighting 380 kg from a trolley to the ground by using a rope which he passes over a fixed horizontal pipe overhead. The man is capable of controlling the motion with a force of 200 N or less on the free end of the rope. Find the minimum number of times the rope must be passed round the pipe if $\mu = 0.22$.

(2.1 turns)

35. A chain drive is used for speed reduction from 240 rpm to 110 rpm. The number of teeth on the driving sprocket is 22. The centre to centre distance between two sprockets is 540 mm and the pitch circle diameter of the driven sprocket is 480 mm. Determine the number of teeth on the driven sprocket, pitch and the length of the chain.

(48, 31.4 mm, 2.21 m)

10 GEARS

Introduction

Gears are used to transmit motion from one shaft to another or between a shaft and a slide. This is accomplished by successively engaging teeth.

Gears use no intermediate link or connector and transmit the motion by direct contact. In this method, the surfaces of two bodies make a tangential contact. The two bodies have either a rolling or a sliding motion along the tangent at the point of contact. No motion is possible along the common normal as that will either break the contact or one body will tend to penetrate into the other.

If power transmitted between two shafts is small, motion between them may be obtained by using two plain cylinders or discs 1 and 2 as shown in Fig. 10.1. If there is no slip of one surface relative to the other, a definite motion of 1 can be transmitted to 2 and vice-versa. Such wheels are termed as *friction wheels*. However, as the power transmitted increases, slip occurs between the discs and the motion no longer remains definite.

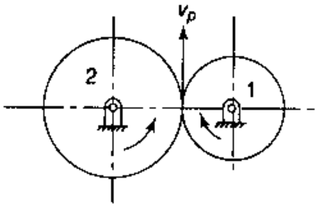


Fig. 10.1

Assuming no slipping of the two surfaces, the following kinematic relationship exists for their linear velocity:

$$v_p = \omega_1 r_1 = \omega_2 r_2$$
$$= 2\pi N_1 r_1 = 2\pi N_2 r_2$$

or
$$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{r_2}{r_1} \tag{10.1}$$

where N = angular velocity (rpm)
ω = angular velocity (rad/s)
r = radius of the disc

Subscripts 1 and 2 represent discs 1 and 2 respectively.

The relationship shows that the speeds of the two discs rolling together without slipping are inversely proportional to the radii of the discs.

To transmit a definite motion of one disc to the other or to prevent slip between the surfaces, projections and recesses on the two discs can be made which can mesh with each other. This leads to the formation of teeth on the discs and the motion between the surfaces changes from rolling to sliding. The discs with teeth are known as *gears* or *gear wheels*.

It is to be noted that if the disc 1 rotates in the clockwise direction, 2 rotates in the counter-clockwise direction and vice-versa.

Although large velocity ratios of the driving and the driven members have been obtained by the use of gears, practically, it is limited to 6 for spur gears and 10 for helical and herringbone gears. To obtain large reductions, two or more pairs of gears are used.

10.1 CLASSIFICATION OF GEARS

Gears can be classified according to the relative positions of their shaft axes as follows:

1. Parallel Shafts

Regardless of the manner of contact, uniform rotary motion between two parallel shafts is equivalent to the rolling of two cylinders, assuming no slipping. Depending upon the teeth of the equivalent cylinders, i.e., straight or helical, the following are the main types of gears to join parallel shafts:

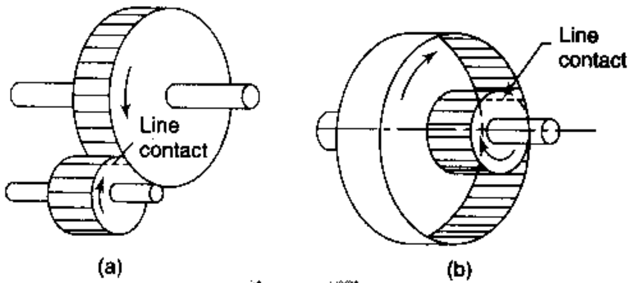


Fig. 10.2

Spur Gears They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load [Fig. 10.2(a)].

At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axes of rotation. This results in sudden application of the load, high impact stresses and excessive noise at high speeds.

Further, if the gears have external teeth on the outer surface of the cylinders, the shafts rotate in the opposite direction [Fig. 10.2(a)]. In an internal spur gear, the teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion (smaller gear) only and the two shafts rotate in the same direction as shown in [Fig. 10.2(b)].

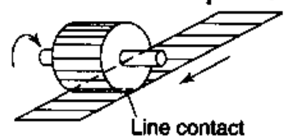


Fig. 10.3

Spur Rack and Pinion Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is a plane (Fig. 10.3). The spur rack and pinion combination converts rotary motion into translatory motion, or vice-versa. It is used in a lathe in which the rack transmits motion to the saddle.

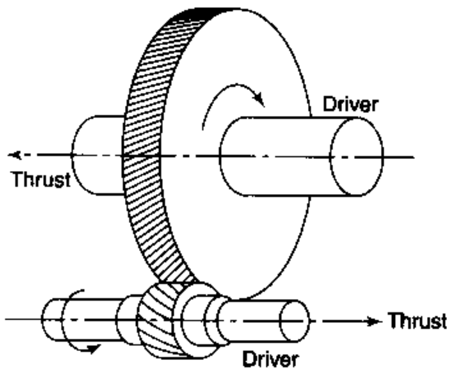


Fig. 10.4

Helical Gears or Helical Spur Gears In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle, but have teeth of opposite hands (Fig. 10.4).

At the beginning of engagement, contact occurs only at the point of leading edge of the curved teeth. As the gears rotate, the contact extends along a diagonal line across the teeth. Thus, the load application is gradual which results in low impact stresses and reduction in noise. Therefore, the helical gears can be used at higher velocities than the spur gears and have greater load-carrying capacity.

Helical gears have the disadvantage of having end thrust as there is a force component along the gear axis. The bearings and the assemblies mounting the helical gears must be able to withstand thrust loads.

Double-helical and Herringbone Gears A *double-helical* gear is equivalent to a pair of helical gears secured together, one having a right-hand helix and the other a left-hand helix. The teeth of the two rows are separated by a groove used for tool run out. Axial thrust which occurs in case of single-helical gears is eliminated in double-helical gears. This is because the axial thrusts of the two rows of teeth cancel each other out. These can be run at high speeds with less noise and vibrations.

If the left and the right inclinations of a double-helical gear meet at a common apex and there is no groove in between, the gear is known as *herringbone* gear (Fig. 10.5).

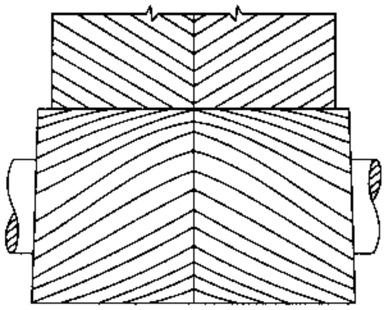


Fig. 10.5

2. Intersecting Shafts

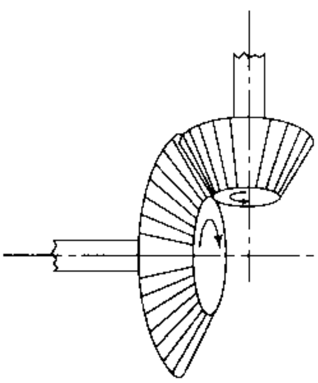


Fig. 10.6

Kinematically, the motion between two intersecting shafts is equivalent to the rolling of two cones, assuming no slipping. The gears, in general, are known as *bevel* gears.

When teeth formed on the cones are straight, the gears are known as *straight bevel* and when inclined, they are known as *spiral* or *helical bevel*.

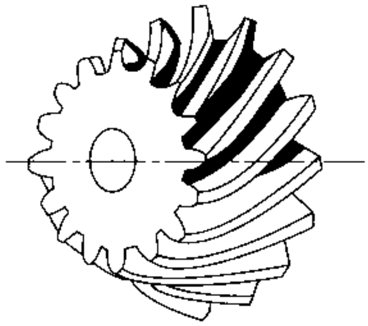


Fig. 10.7

Straight Bevel Gears The teeth are straight, radial to the point of intersection of the shaft axes and vary in cross section throughout their length. Usually, they are used to connect shafts at right angles which run at low speeds (Fig. 10.6). Gears of the same size and connecting two shafts at right angles to each other are known as *mitre* gears.

At the beginning of engagement, straight bevel gears make the line contact similar to spur gears. There can also be *internal bevel* gears analogous to internal spur gears.

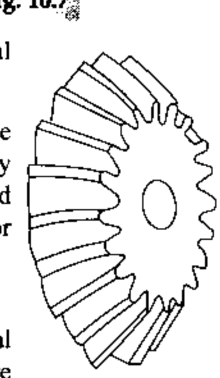


Fig. 10.8

Spiral Bevel Gears When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as *spiral bevels* or *helical bevels* (Fig. 10.7). They are smoother in action and quieter than straight tooth bevels as there is gradual load application and low impact stresses. Of course, there exists an axial thrust calling for stronger bearings and supporting assemblies.

These are used for the drive to the differential of an automobile.

Zerol Bevel Gears Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zerol bevel gears (Fig. 10.8). Their tooth action and the end thrust are the same as that of straight bevel gears and, therefore, can be used in the same mountings. However, they are quieter in action than the straight bevel type as the teeth are curved.

3. Skew Shafts

In case of parallel and intersecting shafts, a uniform rotary motion is possible by pure rolling contact. But in case of skew (non-parallel, non-intersecting) shafts, this is not possible.

Observe a hyperboloid shown in Fig. 10.9(a). It is a surface of revolution generated by a skew line AB revolving around an axis O-O in another plane, keeping the angle ψ_1 between them as constant. The minimum distance between AB and O-O is the common perpendicular CD which is also the radius of the gorge or throat of the hyperboloid.

As the generating element of a hyperboloid is a straight line, two hyperboloids can contact each other on a line common to their respective generating element, e.g., AB can be the generating element of the two hyperboloids [Fig. 10.9(b)]. Further, if the two mating hyperboloids are of limited width and have the rolling motion only, then contact length of their generators will go on diminishing and soon the two could be separated. In other words, if it is desired that the two hyperboloids touch each other on the entire length of AB as they roll, they must have some sliding motion parallel to the line of contact. Thus, if the two hyperboloids rotate on their respective axes, the motion between them would be a combination of rolling (normal to the line of contact) and sliding action (parallel to the line of contact). Teeth are cut on the hyperboloid surfaces parallel to the line of contact to form gears.

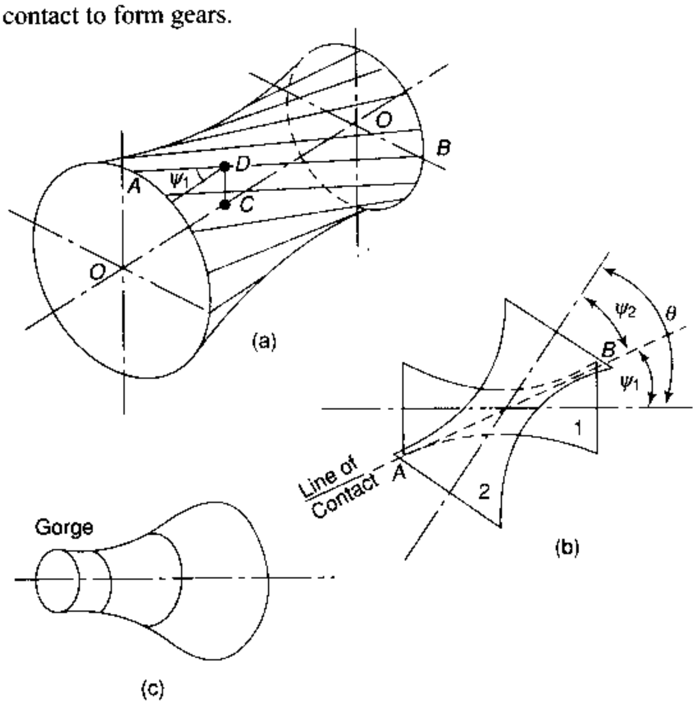


Fig. 10.9

Angle between the two shafts will be equal to the sum of the angles of generation of the two hyperboloids.

$$\theta = \psi_1 + \psi_2 \tag{10.2}$$

The minimum perpendicular distance between the two shafts is the sum of the gorge (throat) radii.

In practice, due to manufacturing difficulties, only portions of the hyperboloids are used to transmit motion between the skew shafts and that too with approximations as given below:

1. A short segment at the gorge is approximated to a cylinder and the corresponding gear is known as *helical* or *crossed-helical* or *spiral* gear [Fig. 10.9(c)]. The contact between the two gears is concentrated at a point which limits the capacity.

   For skew shafts with a 90° angle between them where high-speed ratios are to be achieved, the helix angle of the pinion (small gear) increases. When the angle exceeds 60°–65° and the number of teeth is less than 3–4, the high-speed pinion is known as *worm* and the mating helical gear as the *gear*.

2. Gears using an end portion of the hyperboloid are known as *hypoid* gears.
Thus, the main types of gears used for skew shafts are the following:

Crossed helical Gears The use of crossed-helical gears or spiral gears is limited to light loads. By a suitable choice of helix angle for the mating gears, the two shafts can be set at any angle (Fig. 10.10).

These gears are used to drive feed mechanisms on machine tools, camshafts and oil pumps on small IC engines, etc.

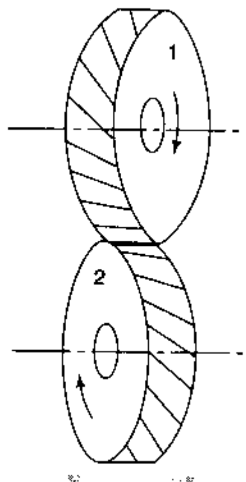


Fig. 10.10

Worm Gears Worm gear is a special case of a spiral gear in which the larger wheel, usually, has a hollow or concave shape such that a portion of the pitch diameter of the other gear is enveloped on it. The smaller of the two wheels is called the worm which also has a large spiral angle.

The shafts may have any angle between them, but normally it is 90°. At least, one tooth of the worm must make a complete turn around the pitch cylinder and thus forms the screw thread. The sliding velocity of a worm gear is higher as compared to other types of gears.

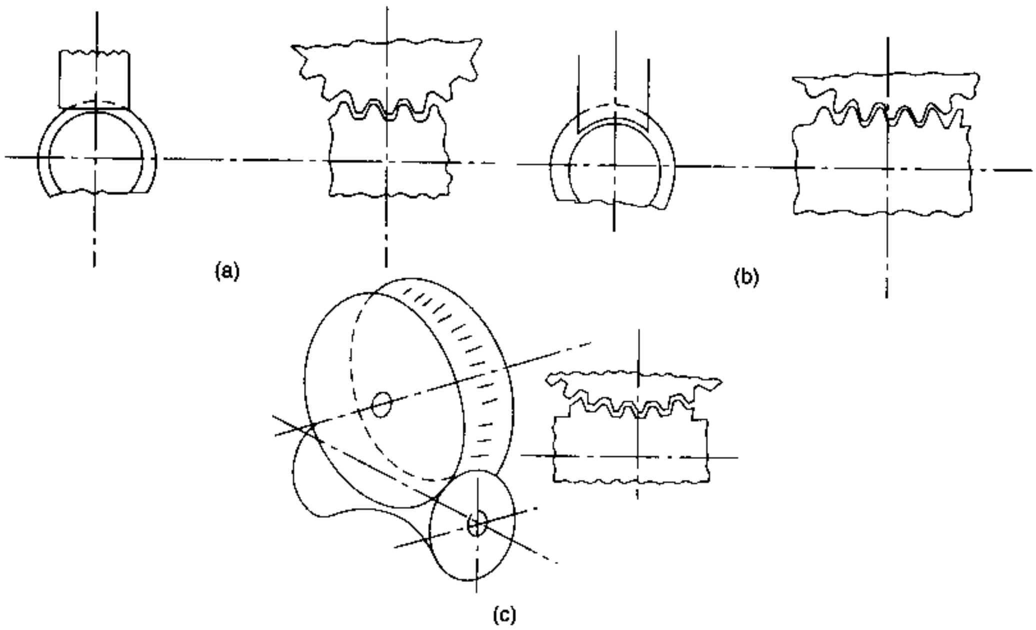


Fig. 10.11

Worm gears are made in the following forms:

1. *Non-throated (Fig. 10.11a)* The contact between the teeth is concentrated at a point.
2. *Single-throated (Fig. 10.11b)* Gear teeth are curved to envelop the worm. There is line contact between the teeth.
3. *Double-throated (Fig. 10.11c)* There is area contact between the teeth. A worm may be cut with a single- or a multiple-thread cutter.

Hypoid Gears As mentioned earlier, hypoid gears are approximations of hyperboloids though they look like spiral gears [Fig. 10.12(a)]. A hypoid pinion is larger and stronger than a spiral bevel pinion. A hypoid pair has a quiet and smooth action. Moreover, the shafts can pass each other so that bearings can be used on both sides of the gear and the pinion [Fig. 10.12(b)].

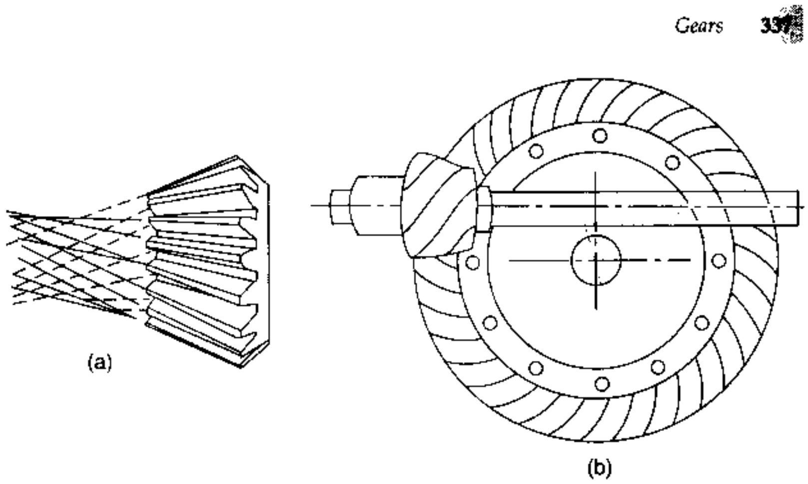


Fig. 10.12

There is continuous pitch line contact of the two mating hypoid gears while in action and they have larger number of teeth in contact than straight-tooth bevel gears. These can wear well if properly lubricated.

10.2 GEAR TERMINOLOGY

Various terms used in the study of gears have been explained below:

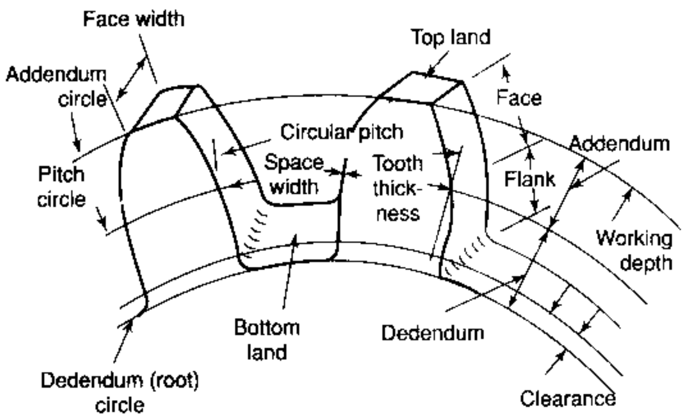


Fig. 10.13

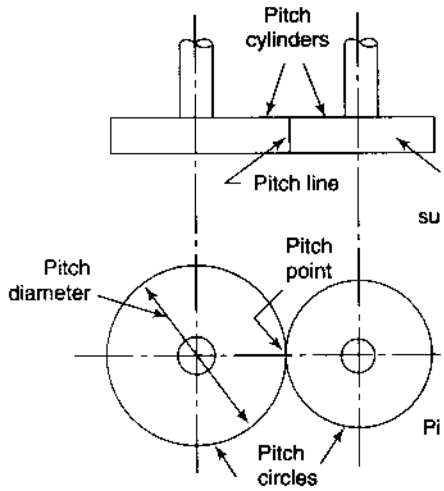


Fig. 10.14

1. Refer Figs 10.13 and 10.14.

   (a) *Pitch Cylinders* Pitch cylinders of a pair of gears in mesh are the imaginary friction cylinders, which by pure rolling together, transmit the same motion as the pair of gears.

   (b) *Pitch Circle* It is the circle corresponding to a section of the equivalent pitch cylinder by a plane normal to the wheel axis.

(c) *Pitch Diameter* It is the diameter of the pitch cylinder.

(d) *Pitch Surface* It is the surface of the pitch cylinder.

(e) *Pitch Point* The point of contact of two pitch circles is known as the pitch point.

(f) *Line of Centres* A line through the centres of rotation of a pair of mating gears is the line of centres.

(g) *Pinion* It is the smaller and usually the driving gear of a pair of mated gears.

2. (a) *Rack* It is a part of a gear wheel of infinite diameter (Fig. 10.15).

(b) *Pitch Line* It is a part of the pitch circle of a rack and is a straight line (Fig. 10.15).

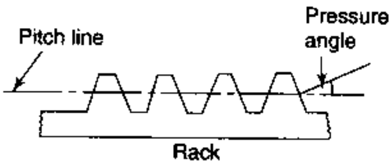


Fig. 10.15

3. *Pitch* It is defined as follows:

(a) *Circular Pitch (p)* It is the distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth (Fig. 10.13).

$$p = \frac{\pi d}{T}$$

where p = circular pitch
d = pitch diameter
T = number of teeth

As the expression for p involves π, an indeterminate number, p, cannot be expressed precisely. The angle subtended by the circular pitch at the centre of the pitch circle is known as the *pitch angle* (y).

(b) *Diametral Pitch (P)* It is the number of teeth per unit length of the pitch circle diameter in inches.

$$P = \frac{T}{d}$$

The limitations of the diametral pitch is that it is not in terms of units of length, but in terms of teeth per unit length.
Also, it can be seen that

$$pP = \frac{\pi d}{T} \cdot \frac{T}{d} = \pi$$

The term *diametral pitch* is not used in SI units.

(c) *Module (m)* It is the ratio of the pitch diameter in mm to the number of teeth. The term is used in SI units in place of diametral pitch.

$$m = \frac{d}{T}$$

Also,

$$p = \frac{\pi d}{T} = \pi m$$

Pitch of two mating gears must be same.

4. (a) *Gear Ratio (G)* It is the ratio of the number of teeth on the gear to that on the pinion.

$$G = \frac{T}{t}$$

where T = number of teeth on the gear

t = number of teeth on the pinion.

(b) *Velocity Ratio (VR)* The velocity ratio is defined as the ratio of the angular velocity of the follower to the angular velocity of the driving gear.

Let d = pitch diameter

T = number of teeth

ω = angular velocity (rad/s)

N = angular velocity (rpm)

Subscript 1 = driver

2 = follower

$$\text{VR} = \frac{\text{angular velocity of follower}}{\text{angular velocity of driver}}$$

$$= \frac{\omega_2}{\omega_1}$$

$$= \frac{N_2}{N_1} \qquad (\omega = 2\,\pi N)$$

$$= \frac{d_1}{d_2} \qquad (\because\ \pi d_1 N_1 = \pi d_2 N_2)$$

$$= \frac{T_1}{T_2} \qquad \left(p = \frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2}\right) \qquad (10.3)$$

5. Refer to Fig. 10.13.

(i) (a) *Addendum Circle* It is a circle passing through the tips of teeth.

(b) *Addendum* It is the radial height of a tooth above the pitch circle. Its standard value is one module.

(c) *Dedendum* or *Root Circle* It is a circle passing through the roots of the teeth.

(d) *Dedendum* It is the radial depth of a tooth below the pitch circle. Its standard value is $1.157\,m$.

(e) *Clearance* Radial difference between the addendum and the dedendum of a tooth. Thus,

Addendum circle diameter = $d + 2m$

Dedendum circle diameter = $d - 2 \times 1.157\ m$

Clearance = $1.157\ m - m$

= $0.157m$

(ii) (a) *Full Depth of Teeth* It is the total radial depth of the tooth space.

Full depth = Addendum + Dedendum

(b) *Working Depth of Teeth* The maximum depth to which a tooth penetrates into the tooth space of the mating gear is the working depth of teeth.

Working depth = Sum of addendums of the two gears.

(c) *Space Width* It is the width of the tooth space along the pitch circle.

(d) *Tooth Thickness* It is the thickness of the tooth measured along the pitch circle.

(e) *Backlash* It is the difference between the space width and the tooth thickness along the pitch circle. Backlash = Space width – Tooth thickness

(f) *Face Width* The length of the tooth parallel to the gear axis is the face width.

(iii) (a) *Top Land* It is the surface of the top of the tooth.

(b) *Bottom Land* The surface of the bottom of the tooth between the adjacent fillets.

(c) *Face* Tooth surface between the pitch circle and the top land.

(d) *Flank* Tooth surface between the pitch circle and the bottom land including fillet.

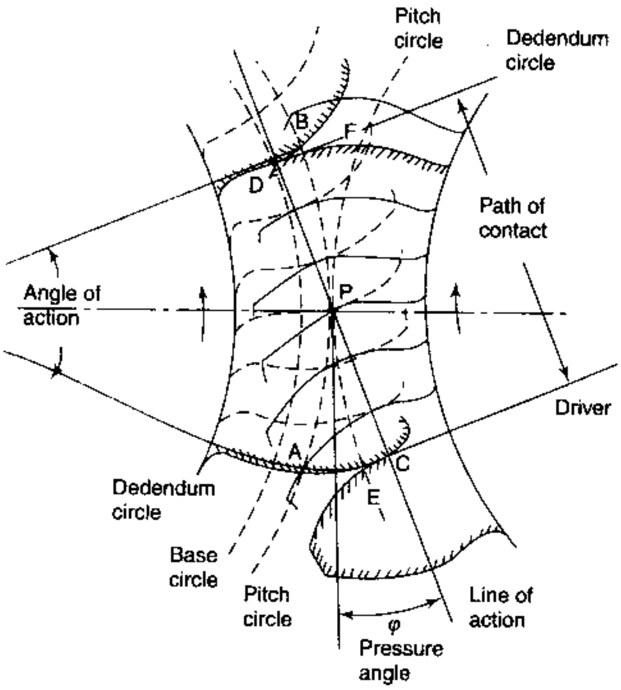


Fig. 10.16

(e) *Fillet* It is the curved portion of the tooth flank at the root circle.

6. Refer Fig. 10.16

(i) (a) *Line of Action or Pressure Line* The force, which the driving tooth exerts on the driven tooth, is along a line from the pitch point to the point of contact of the two teeth. This line is also the common normal at the point of contact of the mating gears and is known as the line of action or the pressure line.

(b) *Pressure Angle or Angle of Obliquity* (φ) The angle between the pressure line and the common tangent to the pitch circles is known as the pressure angle or the angle of obliquity. For more power transmission and lesser pressure on the bearings, the pressure angle must be kept small. Standard pressure angles are 20° and 25°. Gears with 14.5° pressure angles have become almost obsolete.

(ii) (a) *Path of Contact or Contact Length* The locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the path of contact or the contact length. It is *CD* in the figure. The pitch point *P* is always one point on the path of contact. It can be subdivided as follows:

Path of Approach Portion of the path of contact from the beginning of engagement to the pitch point, i.e., the length *CP*.

Path of Recess Portion of the path of contact from the pitch point to the end of engagement, i.e., length *PD*.

(b) *Arc of Contact* The locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the arc of contact. In Fig. 10.16, *APB* or *EPF* is the arc of contact.

It has also been divided into sub-portions.

Arc of Approach It is the portion of the arc of contact from the beginning of engagement to the pitch point, i.e., length *AP* or *EP*.

Arc of Recess The portion of the arc of contact from the pitch point to the end of engagement is the arc of recess, i.e., length *PB* or *PF*.

(c) *Angle of Action (δ)* It is the angle turned by a gear from the beginning of engagement to the end of engagement of a pair of teeth, i.e., the angle turned by arcs of contact of respective gear wheels.

Similarly, the angle of approach (α) and angle of recess (β) can be defined.

$$\delta = \alpha + \beta$$

The angle will have different values for the driving and the driven gears.

7. *Contact Ratio* It is the angle of action divided by the pitch angle, i.e.,

$$\text{Contact ratio} = \frac{\delta}{\gamma} = \frac{\alpha + \beta}{\gamma}$$

As the angle of action is the angle subtended by arc of contact and the pitch angle is the angle subtended by the circular pitch at the centre of the pitch circle, contact ratio is also the ratio of the arc of contact to the circular pitch, i.e.,

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

Example* 10.1** ***Two spur gears have a velocity ratio of 1/3. The driven gear has 72 teeth of 8 mm module and rotates at 300 rpm. Calculate the number of teeth and the speed of the driver. What will be the pitch line velocities?

Solution $T_2 = 72$; $VR = 1/3$; $N_2 = 300$ rpm; $m = 8$ mm

(i) $VR = \frac{N_2}{N_1} = \frac{T_1}{T_2} = \frac{1}{3}$ or $\frac{300}{N_1} = \frac{1}{3}$

or $N_1 = 900$ rpm

Also $\frac{T_1}{72} = \frac{1}{3}$ or $T_1 = 24$

(ii) Pitch line velocity, $v_p = \omega_1 r_1$ or $\omega_2 r_2$

$$= 2\pi N_1 \times \frac{d_1}{2} \text{ or } 2\pi N_2 \times \frac{d_2}{2}$$

$$= 2\pi N_1 \times \frac{mT_1}{2} \text{ or } 2\pi N_2 \times \frac{mT_2}{2}$$

$$= 2\pi \times 900 \times \frac{8 \times 24}{2} \text{ or } 2\pi \times 300 \times \frac{8 \times 72}{2}$$

= 542 867 mm/minute

= 9047.8 mm/s or 9.0478 m/s

Example* 10.2** ***The number of teeth of a spur gear is 30 and it rotates at 200 rpm. What will be its circular pitch and the pitch line velocity if it has a module of 2 mm?

Solution $T = 30$; $m = 2$ mm; $N = 200$ rpm

$p = \pi m = \pi \times 2 = \underline{6.28 \text{ mm}}$

$$v_p = \omega r = 2\pi N \times \frac{d}{2} = 2\pi N \times \frac{mT}{2}$$

$= \pi \times 200 \times 2 \times 30$

$= 37699$ mm / min $= 628.3$ mm/s

10.3 LAW OF GEARING

The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio between two gears. Figure 10.17 shows two bodies 1 and 2 representing a portion of the two gears in mesh.

A point C on the tooth profile of the gear 1 is in contact with a point D on the tooth profile of the gear 2. The two curves in contact at points C or D must have a common normal at the point. Let it be $n - n$.

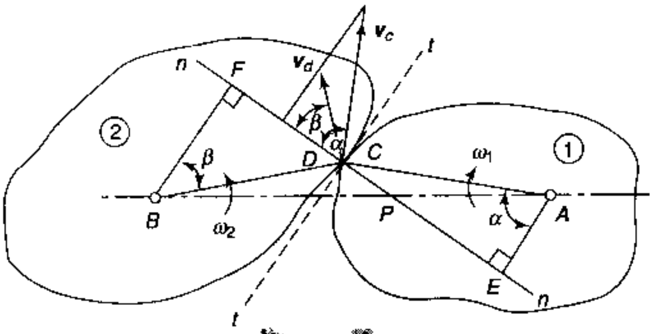


Fig. 10.17

Let ω_1 = instantaneous angular velocity of the gear 1 (clockwise)

ω_2 = instantaneous angular velocity of the gear 2 (counter-clockwise)

v_c = linear velocity of C

v_d = linear velocity of D

Then $v_c = \omega_1.AC$ in a direction perpendicular to AC or at an angle α to $n - n$.

$v_c = \omega_2.BD$ in a direction perpendicular to BD or at an angle β to $n - n$.

Now, if the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent $t - t$. The relative motion between the surfaces along the common normal $n - n$ must be zero to avoid the separation, or the penetration of the two teeth into each other.

Component of v_c along $n - n = v_c \cos\alpha$

Component of v_d along $n - n = v_d \cos\beta$

Relative motion along $n - n = v_c \cos\alpha - v_d \cos\beta$

Draw perpendiculars AE and BF on $n - n$ from points A and B respectively. Then $\angle CAE = \alpha$ **and** $\angle DBF = \beta$. For proper contact,

$$v_c \cos \alpha - v_d \cos \beta = 0$$

or $$\omega_1 AC \cos \alpha - \omega_2 BD \cos \beta = 0$$

or $$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

or $$\omega_1 AE - \omega_2 BF = 0$$

or $$\frac{\omega_1}{\omega_2} = \frac{BF}{AE}$$

$$= \frac{BP}{AP}$$

[∵ ΔAEP and BEP are similar]

Thus, it is seen that the centre line AB is divided at P by the common normal in the inverse ratio of the angular velocities of the two gears. If it is desired that the angular velocities of two gears remain constant, the common normal at the point of contact of the two teeth should always pass through a fixed point P which divides the line of centres in the inverse ratio of angular velocities of two gears.

As seen earlier, P is also the point of contact of two pitch circles which divides the line of centres in the inverse ratio of the angular velocities of the two circles and is the pitch point.

Thus, for constant angular velocity ratio of the two gears, the common normal at the point of contact of the two mating teeth must pass through the pitch point.

Also, as the Δs AEP and BFP are similar,

$$\frac{BP}{AP} = \frac{FP}{EP}$$

or

$$\frac{\omega_1}{\omega_2} = \frac{FP}{EP} \quad \text{or} \quad \omega_1 EP = \omega_2 FP \tag{10.4}$$

10.4 VELOCITY OF SLIDING

If the curved surfaces of the two teeth of the gears 1 and 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent $t - t$ at C or D (Fig. 10.17).

Component of v_c along $t - t = v_c \sin\alpha$

Component of v_d along $t - t = v_d \sin\alpha$

Velocity of sliding $= v_c \sin\alpha - v_d \sin\beta$

$$= \omega_1 AC \frac{EC}{AC} - \omega_2 BD \frac{FD}{BD}$$

$$= \omega_1 EC - \omega_2 FD$$

$$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$$

$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC$$

(C and D are the coinciding points)

$$= (\omega_1 + \omega_2)\, PC + \omega_1 EP - \omega_2 FP$$

$$= (\omega_1 + \omega_2)\, PC \qquad [\omega_1 EP = \omega_2 FP, \text{ Eq. (10.4)}]$$

= sum of angular velocities × distance between the pitch point and the point of contact

10.5 FORMS OF TEETH

Two curves of any shape that fulfill the law of gearing can be used as the profiles of teeth. In other words, an arbitrary shape of one of the mating teeth can be taken and applying the law of gearing the shape of the other can be determined. Such gear are said to have *conjugate* teeth. However, it will be very difficult to manufacture such gears and the cost will be high. Moreover, on wearing, it will be very difficult to replace them with the available gears. Thus, there arises the need to standardize gear teeth.

Common forms of teeth that also satisfy the law of gearing are

1. Cycloidal profile teeth
2. Involute profile teeth

10.6 CYCLOIDAL PROFILE TEETH

In this type, the faces of the teeth are epicycloids and the flanks are the hypocycloids.

A *cycloid* is the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.

An *epicycloid* is the locus of a point on the circumference of a circle that rolls without slipping on the circumference of another circle.

A *hypocycloid* is the locus of a point on the circumference of a circle that rolls without slipping inside the circumference of another circle.

The formation of a cycloidal tooth has been shown in Fig. 10.18. A circle H rolls inside another circle APB (pitch circle). At the start, the point of contact of the two circles is at A. As the circle H rolls inside the pitch circle, the locus of the point A on the circle H traces the path ALP which is a hypocycloid. A small portion of this curve near the pitch circle is used for the flank of the tooth.

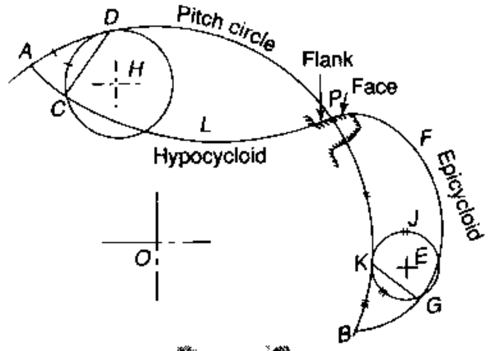


Fig. 10.18

A property of the hypocycloid is that at any instant, the line joining the generating point (A) with the point of contact of the two circles is normal to the hypocycloid, e.g., when the circle H touches the pitch circle at D, the point A is at C and CD is normal to the hypocycloid ALP.

Also, Arc AD = Arc CD (on circle H)

In the same way, if the circle E rolls outside the pitch circle, starting from P, an epicycloid PFB is obtained. Similar to the property of a hypocycloid, the line joining the generating point with the point of contact of the two circles is a normal to the epicycloid, e.g., when the circle E touches the pitch circle at K, the point P is at G and GK is normal to the epicycloid PFB.

Arc PK = Arc KJG (on circle E)

or Arc BK = Arc KG (on circle E)

A small portion of the curve near the pitch circle is used for the face of the tooth.

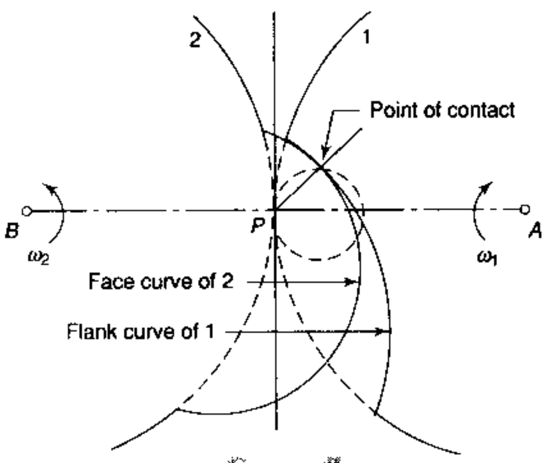


Fig. 10.19

Meshing of Teeth

During meshing of teeth, the face of a tooth on one gear is to mesh with the flank of another tooth on the other gear. Thus, for proper meshing, it is necessary that the diameter of the circle generating face of a tooth (on one gear) is the same as the diameter of the circle generating flank of the meshing tooth (on another gear); the pitch circle being the same in the two cases (Fig. 10.19).

Of course, the face and the flank of a tooth of a gear can be generated by two circles of different diameters. However, for interchangeability, the faces and flanks of both the teeth in the mesh are generated by the circles of the same diameter.

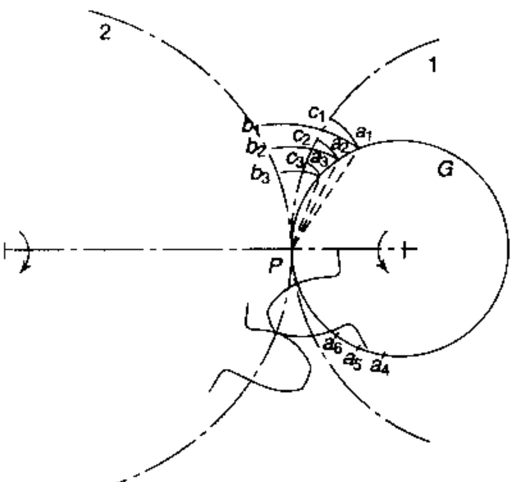


Fig. 10.20

Consider a generating circle G rolling outside the pitch circle of the gear 2 (Fig. 10.20). It will generate epicycloid, a portion of which is the face of tooth on the gear. Now this face is to mesh with the flank of a tooth on the gear 1. This flank will be a portion of the hypocycloid which can be generated by rolling the same generating circle G inside the pitch circle of the gear 1.

a_1 is the generating point for the two curves a_1b_1 (epicycloid) and a_1c_1 (hypocycloid). a_1b_1 is generated when the circle G moves in the clockwise direction on the pitch circle of the gear 2 and at the start a_1 coincides with b_1. a_1c_1 is generated when the circle G moves clockwise inside the pitch circle of the gear 2, and in the beginning a_1 coincides with c_1.

The two pitch circles touch each other at P (pitch point). When the generating circle G touches the pitch circle 2 at P, the generating point of the epicycloid is at a_1 and a_1P is normal to the face of tooth on the gear 2. Similarly, when G touches the pitch circle 1 at P, the generating point of the hypocycloid is again at a_1 and a_1P is also normal to the flank of tooth on the gear 1. Thus, if at an instant, a_1P is the common normal to the two profiles of the meshing teeth, the teeth must touch each other tangentially.

According to the law of gearing, the common normal at the point of contact of two mating profiles of the teeth must pass through a fixed point which is also the pitch point. The above discussion shows that the law of gearing is fulfilled in case of cycloidal teeth.

After a little while, let the point of contact of the two mating gears be at a_2. This point is on the generating circle G and if b_2 is considered the start of the epicycloid a_2b_2, and c_2 is considered the start of the hypocycloid a_2c_2 then a_2P will be normal to the two curves a_2b_2 and a_2c_2.

But as the two curves a_1b_1 and a_2b_2 are generated by the same circle rolling outside the same pitch circle, the two curves must be similar. Thus, a_2b_2 can be a portion of the curve a_1b_1. Similarly, a_2c_2 can be a portion of the curve a_1c_1.

Thus, in case of cycloidal teeth, the points of contact such as $a_1, a_2, a_3, \ldots, P$ lie on the generating circle G.

After passing through the point P, the point of contact will shift on the other generating circle. Now, the flank of the tooth of the gear 1 will touch the face of the tooth of the gear 2. Thus, path of contact of cycloidal gears lies on the generating circles.

Path of approach = Arc $a_1a_2a_3P$

Arc of approach = Arc $b_1b_2b_3P$ = Arc $c_1c_2c_3P$

But arc $a_1a_2a_3P$ = Arc $b_1b_2b_3P$ = Arc $c_1c_2c_3P$

Therefore, the path of approach is equal to the arc of approach. In the same way, it can be shown that the path of contact will be equal to the arc of contact.

If the direction of rotation of the driver is reversed, the path of approach will be $a_4, a_5, a_6, \ldots, P$

Observe that in case of cycloidal teeth, the pressure angle varies from the maximum at the beginning of engagement to zero when the point of contact coincides with pitch point P and then again increases to maximum in the reverse direction.

As the common normal to the two meshing curves passes through the pitch point P, uniform rotary motion will be transmitted only as long as the pitch circles are tangent to each other. If the centre distance between the two pitch circles varies, the point P is shifted and the speed of the driven gear would vary.

Since the cycloidal teeth are made up of two curves, it is very difficult to produce accurate profiles. This has rendered this system obsolete.

10.7 INVOLUTE PROFILE TEETH

An *involute* is defined as the locus of a point on a straight line which rolls without slipping on the circumference of a circle. Also, it is the path traced out by the end of a piece of taut cord being unwound from the circumference of a circle. The circle on which the straight line rolls or from which the cord is unwound is known as the *base circle*.

Figure 10.21 shows an involute generated by a line rolling over the circumference of a base circle with centre at *O*. At the start, the tracing point is at *A*. As the line rolls on the circumference of the circle, the path *ABC* traced out by the point *A* is the involute.

Note that as *D* can be regarded as the instantaneous centre of rotation of *B*, the motion of *B* is perpendicular to *BD*. Since *BD* is tangent to the base circle, the normal to the involute is a tangent to the base circle.

A short length *EF* of the involute drawn from *A* can be utilized to make the profile of an involute tooth. The other side *HJ* of the tooth has been taken from the involute drawn from *G* in the reverse direction. The profile of an involute tooth is made up of a single curve, and teeth, usually, are termed as single curve teeth.

Owing to the ease of standardization and manufacture, and low cost of production, the use of involute teeth has become universal by entirely superseding the cycloidal shape. Only one cutter or tool is necessary to manufacture a complete set of interchangeable gears. The cutter is in the form of a rack as all gears will gear with their corresponding rack. Moreover, the cutters of this form can be made to a higher degree of accuracy as the teeth of an involute rack are straight.

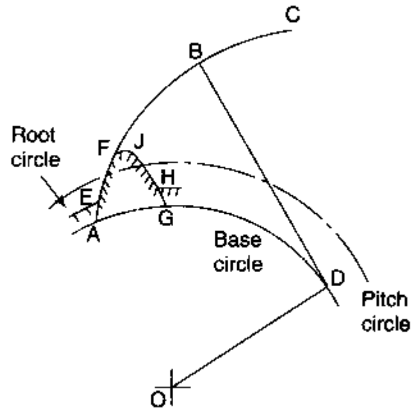


Fig. 10.21

Gear cutter of a milling machine. It cuts involute teeth.

Meshing of Teeth

In Fig. 10.22, two gear wheels 1 and 2 with centres of rotation at *A* and *B* respectively are in contact at *C*. *CE* and *CF* are the tangents to the two base circles 1 and 2 respectively. *t-t* is the common tangent to the two involutes *DC* and *GCH* of the two meshing teeth. The involute *DC* is traced by rolling line *EF* on the base circle of the gear 2 while the involute *GCH* is obtained by rolling line *EF* on the base circle of the gear 1.

Cutter of a hobbing machine. It cuts multiple teeth.

From the property of the involute, the tangent *CF* to the base circle of the gear 2 is normal to the involute *DC* or the tangent *t* – *t*. Similarly, the tangent *CE* to the base circle of the gear 1 is normal to the involute *GC* or the tangent *t* – *t*. As *CE* and *CF* both are normal to the common tangent t–t at the point *C*, *CE* and *CF* lie on a straight line. ECF is thus a straight line.

As the wheel 1 rotates in the clockwise direction, the point of contact C on the involute GCH pushes the involute DC along the line CF. Therefore, the path of contact of the two involute teeth is along the common tangent to the base circles. This common tangent is also the common normal to the two involutes at the point of contact for all positions.

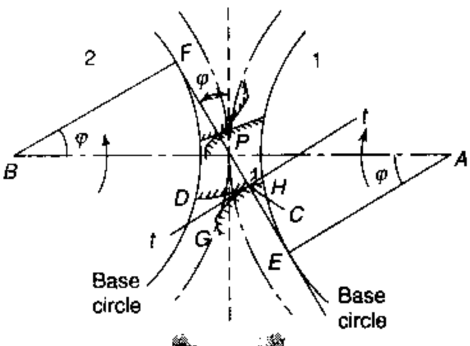


Fig. 10.22

Also, the common normal to the two involutes divides the line of centres of the two gears at P, the pitch point. Thus, the common normal always passes through the pitch point which is the point of contact of two pitch circles.

The line of action in case of involute teeth is along the common normal at the point of contact, which is fixed and is the common tangent to the two base circles. This shows that the pressure angle in this case remains constant throughout the engagement of the two teeth. The usual values of the pressure angles are 14.5°, 20° and 25°.

As EF is tangent to the base circle 1, AE is perpendicular to EF.

AEP is a right-angled triangle.

Also $\angle EAP = \varphi$

$AE = AP \cos \varphi$

Similarly, $BF = BP \cos\varphi$

i.e., [Base circle diameter = Pitch circle diameter × $\cos\varphi$]

$$\text{velocity ratio of gears} = \frac{BP}{AP} = \frac{BF}{AE} = \text{constant}$$

Thus, for a pair of involute gears, the velocity ratio is inversely proportional to the pitch circle diameters as well as base circle diameters.

Any shift in the centres of two gears changes the centre distance. If the involutes are still in contact, the common normal to the two involutes at the point of contact will be the new common tangent to the base circles and its intersection with the line of centres as the new pitch point (Fig. 10.23). It can be judged that the shifting of P does not alter the ratio AP/BP which means the velocity ratio between the two gears remains constant. Of course, in this way there is change in the pressure angle. Altering the centre distance without destroying the correct tooth action is an important property of the involute gears.

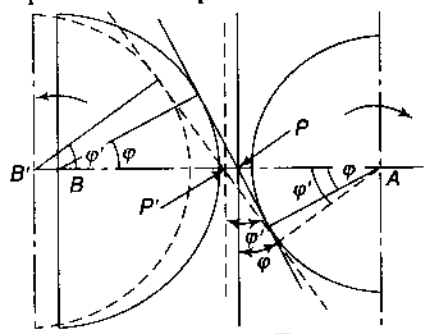


Fig. 10.23

Remember the following in case of involute gears:

1. Points of contact lie on the line of action which is the common tangent to the two base circles.
2. The contact is made when the tip of a tooth of the driven wheel touches the flank of a tooth of the driving wheel and the contact is broken when the tip of the driving wheel touches the flank of the driven wheel.
3. If the direction of angular movement of the wheels is reversed, the points of contact will lie on the other common tangent to the base circles.
4. Initial contact occurs where the addendum circle of the driven wheel intersects the line of action. Final contact occurs at a point where the addendum circle of the driver intersects the line of action.

10.8 INTERCHANGEABLE GEARS

The gears are interchangeable if they are standard ones. It is always a matter of convenience to have gears of standard dimensions which can be replaced easily when they are worn out. The gears are interchangeable if they have

- the same module,
- the same pressure angle,
- the same addendums and dedendums, and
- the same thickness.

A tooth system which relates the various parameters of gears such as pressure angle, addendum, dedendum, tooth thickness, working depth, etc., to attain interchangeability of the gears of all tooth numbers, but of the same pressure angle and pitch is said to be a *standard system*. Usually, the standard cutters are available for their manufacture.

In Table 10.1, tooth proportions for completely interchangeable gears are given. They can be used for operation on standard centre distances. The 14.5° pressure angle system has become obsolete now as the size of the gears used to be larger as compared to the gears with higher angles.

Table 10.1

Tooth system	Pressure angle	addendum	dedendum
Full depth	20°	1 m	1.25 m or 1.35 m
	22.5°	1 m	1.25 m or 1.35 m
	25°	1 m	1.25 m or 1.35 m
Stub	20°	0.8 m	1 m

Preferred modules: 1, 1.25, 1.5, 2, 2.5 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 30, 40, 50

10.9 NON-STANDARD GEARS

The term non-standard gears apply to such gears as are modified by changing some standard parameters like pressure angle, addendum, tooth depth or centre distance. These changes are made to improve the performance of the gear operation or from the economical point of view.

The recent trend these days is to make the designs of machines as compact as possible to reduce their size and weight which also results in reduction in the costs. Consider a gear set to have a velocity ratio of 4:1. If a pinion of 80 mm pitch diameter is selected for the purpose, the pitch diameter of the gear is 320 mm. Thus, space requirement of the gear is 400 mm. Now, if somehow the pitch diameter of the pinion is reduced by 10 mm, the pitch diameter of the gear is reduced by 40 mm, and the overall reduction in space is 50 mm. Also, the sizes of other components associated with the gear set such as shafts, casings and bearings are also reduced. The only way to have a smaller size of gears is to reduce the number of teeth. However, for a typical type of teeth, it is observed that if the number of teeth is reduced from a certain number, the problems of interference, undercutting and contact ratio hamper the smooth running of the gears. Therefore, the main reason to employ non-standard gears is to prevent interference and undercutting and to maintain a reasonable contact ratio.

It should be remembered that as an involute is generated, its radius of curvature goes on becoming larger and larger, being zero at the base circle. As far as possible, the curve near the base should be avoided because high stresses are developed in the region of sharp curvature.

Centre-distance Modifications The number of teeth on a pinion can be reduced from the minimum allowable number by increasing the centre distance marginally and by changing the tooth proportions and the pressure angle of the gears. A reduction in the interference and improvement in the contact ratio is brought this way. The teeth can be generated with rack cutters of standard pressure angles by displacing the pitch line of the rack from the pitch circle of the gear. This action produces teeth which are thicker than before. As the teeth are cut with a displaced or offset cutter, they will engage at a new pressure angle and at a new centre distance.

Clearance Modifications If the clearance between mating teeth is increased to 0.3 m or 0.4 m instead of the usual value of 0.25 m to have a larger fillet at the root of the tooth, the fatigue strength of the tooth is increased. This way some extra depth is available to smoothen the tooth profile. Interchangeability is not lost this way.

Addendum Modifications In cases where it is not possible to change the centre distances, modifications can be made to the addendum. In such cases, there has to be no change in the pitch circles and the pressure angles. However, the contact region is shifted away from the pinion centre towards the gear centre, decreasing the approach action and increasing the recess action.

Example 10.3 *The following data relate to two meshing gears Velocity ratio* $= \frac{1}{3}$, *Module = 4 mm, Pressure angle = 20°, Centre distance = 200 mm*

Determine the number of teeth and the base circle radius of the gear wheel.

Solution $VR = 1/3$, $\varphi = 20°$, $m = 4$ mm

$C = 200$ mm

(i) $VR = \frac{N_2}{N_1} = \frac{1}{3} = \frac{T_1}{T_2}$ or $T_2 = 3T_1$

and $C = \frac{d_1 + d_2}{2} = \frac{m(T_1 + T_2)}{2}$

or $200 = \frac{4(T_1 + 3T_1)}{2} = 8T_1$

or $T_1 = 25$ and $T_2 = 25 \times 3 = 75$

Number of teeth on gear wheel = 75

(ii) $d_2 = m\,T_2 = 4 \times 75 = 300$ mm

Base circle radius, $d_{b2} = \frac{d_2}{2}\cos\varphi$

$= \frac{300}{2} \times \cos 20° = 141$ mm

10.10 PATH OF CONTACT

Let two gear wheels with centres A and B be in contact (Fig. 10.24).

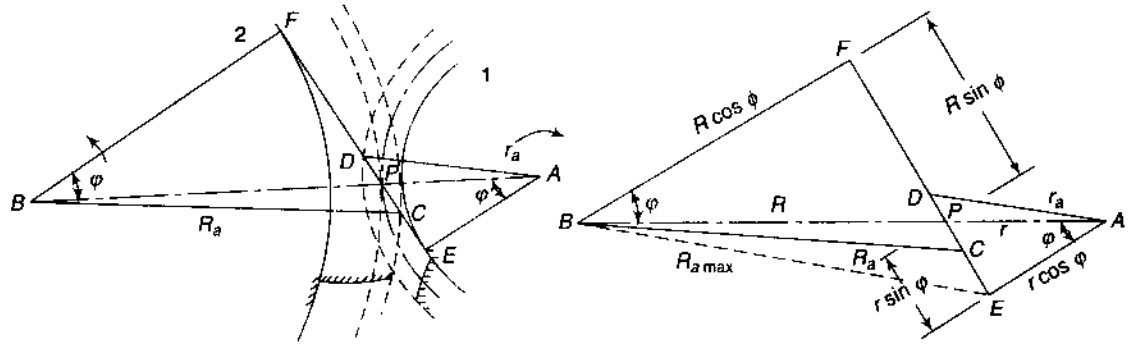


Fig. 10.24

The pinion 1 is the driver and is rotating clockwise. The wheel 2 is driven in the counter-clockwise direction. EF is their common tangent to the base circles.

Contact of the two teeth is made where the addendum circle of the wheel meets the line of action EF, i.e., at C and is broken where the addendum circle of the pinion meets the line of action, i.e., at D. CD is then the path of contact.

Let r = pitch circle radius of pinion

R = pitch circle radius of wheel

r_a = addendum circle radius of pinion

R_a = addendum circle radius of wheel.

Path of contact = path of approach + path of recess

$$CD = CP + PD$$

$$= (CF - PF) + (DE - PE)$$

$$= \left[\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right]$$

$$= \sqrt{R_a^2 - R^2\cos^2\varphi} + \sqrt{r_a^2 - r^2\cos^2\varphi} - (R + r)\sin\varphi \qquad (10.5)$$

Observe that the path of approach can be found if the dimensions of the driven wheel are known. Similarly, the path of recess is known from the dimensions of the driving wheel (pinion).

10.11 ARC OF CONTACT

The arc of contact is the distance travelled by a point on either pitch circle of the two wheels during the period of contact of a pair of teeth.

In Fig.10.25, at the beginning of engagement, the driving involute is shown as GH; when the point of contact is at P, it is shown as JK and when at the end of engagement, it is DL. The arc of contact is $P'P''$ and it consists of the arc of approach $P'P$ and the arc of recess PP''.

Let the time to traverse the arc of approach is t_a. Then

Arc of approach = $P'P$ = Tangential velocity of P' × Time of approach

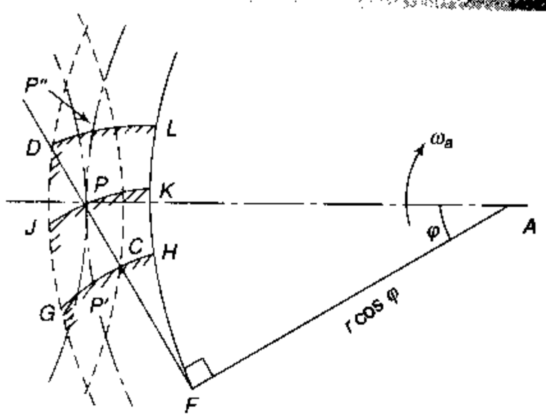


Fig. 10.25

$$= \omega_a r \times t_a \qquad (t_a = \text{time of approach})$$

$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi}t_a$$

$$= (\text{Tang. vel. of } H)t_a\frac{1}{\cos\varphi} \qquad (AF = AH)$$

$$= \frac{\text{Arc } HK}{\cos\varphi}$$

$$= \frac{\text{Arc } FK - \text{Arc } FH}{\cos\varphi}$$

$$= \frac{FP - FC}{\cos\varphi} = \frac{CP}{\cos\varphi}$$

Arc FK is equal to the path FP as the point P is on the generator FP that rolls on the base circle FHK to generate the involute PK. Similarly, arc FH = Path FC.

Arc of recess = PP'' = Tang. vel. of P × Time of recess

$$= \omega_a r \times t_r \qquad (t_r = \text{time of recess})$$

$$= \omega_a (r\cos\varphi) \frac{1}{\cos\varphi} t_r$$

$$= (\text{Tang. vel. of } K)\, t_r \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } KL}{\cos\varphi} = \frac{\text{Arc } FL - \text{Arc } FK}{\cos\varphi}$$

or

$$PP'' = \frac{FD - FP}{\cos\varphi} = \frac{PD}{\cos\varphi}$$

$$\text{Arc of contact} = \frac{CP}{\cos\varphi} + \frac{PD}{\cos\varphi} = \frac{CP + PD}{\cos\varphi} = \frac{CD}{\cos\varphi}$$

or

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi} \qquad (10.6)$$

10.12 NUMBER OF PAIRS OF TEETH IN CONTACT (CONTACT RATIO)

The arc of contact is the length of the pitch circle traversed by a point on it during the mating of a pair of teeth.

Thus, all the teeth lying in between the arc of contact will be meshing with the teeth on the other wheel.

Therefore, the number of teeth in contact, $n = \dfrac{\text{Arc of contact}}{\text{Circular pitch}} = \dfrac{CD}{\cos\varphi}\dfrac{1}{p}$ (10.7)

As the ratio of the arc of contact to the circular pitch is also the *contact ratio*, the number of teeth is also expressed in terms of contact ratio.

For continuous transmission of motion, at least one tooth of one wheel must be in contact with another tooth of the second wheel. Therefore, n must be greater than unity.

If n lies between 1 and 2, the number of teeth in contact at any time will not be less than one and never more than two. If n is between 2 and 3, it is never less than two pairs of teeth and not more than three pairs, and so on. If n is 1.6, one pair of teeth are always in contact whereas two pairs of teeth are in contact for 60% of the time.

Example 10.4 *Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.*

Solution $\varphi = 20°$; $t = T = 48$; $m = 8$ mm;

$$R = r = \frac{mT}{2} = \frac{8 \times 48}{2} = 192 \text{ mm}; \; R_a = r_a$$

Arc of contact = 2.25 × Circular pitch = $2.25\pi m$
= $2.25\pi \times 8$ = 56.55 mm

Path of contact = 56.55 × cos 20° = 53.14 mm

or $\quad (\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi)$

$$+ (\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi) = 53.14$$

or $\quad 2(\sqrt{R_a^2 - 192^2\cos^2 20°} - 192\sin 20°)$

$$= 53.14 \quad \text{or} \quad R_a = 202.6 \text{ mm}$$

Addendum = $R_a - R$ = 202.6 − 192 = 10.6 mm

Example 10.5 *Two involute gears in mesh have 20° pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6 mm. The pitch line velocity is 1.5 m/s and the addendum equal to one module. Determine the angle of action of the pinion (the angle turned by the pinion when one pair of teeth is in the mesh) and the maximum velocity of sliding.*

Solution $\varphi = 20°$; $t = 24$; $m = 6$ mm;

$T = 24 \times 3 = 72$;

$$r = \frac{mt}{2} = \frac{6 \times 24}{2} = 72 \text{ mm};$$

$R = 72 \times 3 = 216$ mm; $r_a = 72 + 6 = 78$ mm ;
$R_a = 216 + 6 = 222$ mm

$$\text{Path of contact} = (\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi)$$
$$+ (\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi)$$

$$= (\sqrt{222^2 - 216^2\cos^2 20°} - 216\sin 20°)$$
$$+ (\sqrt{78^2 - 72^2\cos^2 20°} - 72\sin 20°)$$

= 16.04 + 14.18 = 30.22 mm

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi} = \frac{30.22}{\cos 20°}$$

= 32.16 mm

$$\text{Angle of action} = \frac{\text{Arc of contact}}{r} = \frac{32.16}{72}$$

= 0.4467 rad = 0.4467 × 180/π = 25.59°

Velocity of sliding = $(\omega_p + \omega_g)$ × Path of approach

$$= \left(\frac{v}{r} + \frac{v}{R}\right) \times \text{Path of approach}$$

$$= \left(\frac{1500}{72} + \frac{1500}{216}\right) \times 16.04 = 445.6 \text{ mm/s}$$

Example 10.6 *Two involute gears in a mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addenda on pinion and gear wheels are equal to one module, find the*

(i) contact ratio (the number of pairs of teeth in contact)
(ii) angle of action of the pinion and the gear wheel
(iii) ratio of the sliding to rolling velocity at the
(a) beginning of contact
(b) pitch point
(c) end of contact

Solution $\varphi = 20°$; $T = 57$; $t = 23$; $m = 8$ mm;
addendum = m = 8 mm

$$R = \frac{mT}{2} = \frac{8 \times 57}{2} = 228 \text{ mm};$$

$$R_a = R + m = 228 + 8 = 236 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{8 \times 23}{2} = 92 \text{ mm};$$

$r_a = r + m = 92 + 8 = 100$ mm

(i) $n = \dfrac{\text{Arc of contact}}{\text{Circular pitch}} = \left(\dfrac{\text{Path of contact}}{\cos\varphi}\right)$

$$\times \frac{1}{\pi m} = \frac{\text{Path of approach + Path of recess}}{\cos\varphi \times \pi m}$$

$$= \frac{\left[\begin{array}{l}\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi \\ +\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\end{array}\right]}{\cos\varphi \times \pi m}$$

$$\frac{\left[\begin{array}{l}\sqrt{(236)^2 - (228^2\cos^2 20^\circ} - 228\sin 20^\circ \\ +\sqrt{(100)^2 - (92)^2\cos^2 20^\circ} - 92\sin 20^\circ\end{array}\right]}{\cos 20^\circ \pi \times 8}$$

$$= \frac{20.97 + 18.79}{\cos 20^\circ \times \pi \times 8} = 42.31 \times \frac{1}{\pi \times 8} = \underline{1.68}$$

(ii) Angle of action, $\delta_p = \dfrac{\text{Arc of contact}}{r} = \dfrac{42.31}{92}$

$= 0.46$ rad or $0.46 \times 180/\pi = 26.3^\circ$

$$\delta_g = \frac{\text{Arc of contact}}{R} = \frac{42.31}{228} = 0.1856 \text{ rad}$$

or $0.1856 \times 180/\pi = 10.63^\circ$

(iii) (a) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{(\omega_p + \omega_g) \times \text{Path of approach}}{\text{Pitch line velocity } (= \omega_p \times r)}$$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times 20.97}{\omega_p \times 92} = \underline{0.32}$$

(b) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}} = \dfrac{(\omega_p + \omega_g) \times 0}{\text{Pitch line velocity}} = 0$

(c) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times \text{Path of recess}}{\omega_p \times r}$$

$$= \frac{\left(1 + \frac{23}{57}\right) \times 18.79}{92} = \underline{0.287}$$

Example 10.7 *Two 20° gears have a module pitch of 4 mm. The number of teeth on gears 1 and 2 are 40 and 24 respectively. If the gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the tip of the tooth of gear 2. Take addendum equal to one module.*

Also, find the maximum velocity of sliding.

Solution 1 is the gear wheel and 2 is the pinion.

$\varphi = 20^\circ$; $T = 40$; $N_p = 600$ mm; $t = 24$; $m = 4$ mm

Addendum = 1 module = 4 mm

$$R = \frac{mT}{2} = \frac{4 \times 40}{2} = 80 \text{ mm}; \; R_a = 80 + 4 = 84 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{4 \times 24}{2} = 48 \text{ mm}; \; r_a = 48 + 4 = 52 \text{ mm}$$

$$N_g = N_p \times \frac{t}{T} = 600 \times \frac{24}{40} = 360 \text{ rpm}$$

(i) Let pinion (gear 2) be the driver.

The tip of the driving wheel is in contact with a tooth of the driven wheel at the end of engagement. Thus, it is required to find the path of recess which is obtained from the dimensions of the driving wheel.

Path of recess $= \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$

$= \sqrt{(52)^2 - (48\cos 20^\circ)^2} - 48\sin 20^\circ$

$= 9.458$ mm

Velocity of sliding $= (\omega_p + \omega_g) \times$ Path of recess

$= 2\pi(N_p + N_g) \times 9.458$

$= 2\pi(600 + 360) \times 9.458$

$= 57\,049$ mm/min

$= \underline{950.8 \text{ mm/s}}$

(ii) In case the gear wheel is the driver, the tip of the pinion will be in contact with the flank of a tooth of the gear wheel at the beginning of contact. Thus, it is required to find the distance of the point of contact from the pitch point, i.e., path of approach. The path of approach is found from the dimensions of the driven wheel which is again pinion.

Thus, path of approach

$= \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$

= 9.458 mm, as before

and velocity of sliding = 950.8 mm/s

Thus, it is immaterial whether the driver is the gear wheel or the pinion, the velocity of sliding is the same when the contact is at the tip of the pinion.

The maximum velocity of sliding will depend upon the larger path considering any of the wheels to be the driver.

Consider pinion to be the driver.

Path of recess = 9.458 mm

Path of approach

$= \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi$

$= \sqrt{(84)^2 - (80\cos 20°)} - 80\sin 20°$

= 10.117 mm

This is also the path of recess if the wheel becomes the driver

Maximum velocity of sliding

$= (\omega_p + \omega_g) \times$ Maximum path

$= 2\pi(600 + 360) \times 10.117$

= 61024 mm/min

= <u>1017.1 mm/s</u>

10.13 INTERFERENCE IN INVOLUTE GEARS

Power transmission through a pair of teeth is along the line of action or the common normal to the two involutes at the point of contact. The common normal is also a common tangent to the two base circles and passes through the pitch point. At any instant, the portions of the tooth profiles which are in contact must be involutes so that the line of action does not deviate. If any of the two surfaces is not an involute, the two surfaces would not touch each other tangentially and the transmission of power would not be proper. Mating of two non-conjugate (non-involute) teeth is known as *interference* because the two teeth do not slide properly and thus rough action and binding occurs. Owing to non-involute profile, the contacting teeth have different velocities which can lock the two gears.

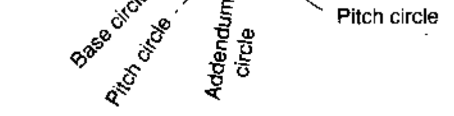


Fig. 10.26

Figure 10.26 shows two gears in mesh. If the pinion is the driver, the line of action will be along EF which is the common tangent to base circles of the two gears. Let the addendum radius of the wheel be BC and that of pinion, AD. The teeth on the pinion and wheel are engaged at C and are disengaged at D. Now, if the radius of the addendum circle of the pinion is increased, the point D shifts along PF towards F and the point D coincides with F when the radius is equal to AF. Any further increase in the value of this radius will result in shifting the point of contact inside the base circle of the wheel. Since an involute can exist only outside the base circle, therefore, any profile of teeth inside the base circle will be of a non-involute type. Usually, a radial profile is adopted for this portion. Thus, the profiles in such a case cannot be tangent to each other and the tip of the pinion would try to dig out the flank of the tooth of the wheel. Therefore, interference occurs in the mating of two gears.

Similarly, if the addendum radius of the wheel is made greater than BE, the tip of the wheel tooth will be in contact with a portion of the non-involute profile of the pinion tooth for some of the engagement. The conclusion is that to have no interference of the teeth, the addendum circles of the wheel and the pinion must intersect the line of action between E and F. The points E and F are called *interference points*.

Note that to avoid interference, the limiting value of the addendum of the wheel is GE whereas that of the pinion is HF and the latter is clearly greater than the former. Thus, if the addenda of the wheel and the pinion are to be equal, the addendum circle of the wheel passes through the limiting point E on the line of action before the addendum circle of the pinion passes through the limiting point F on the same line. Thus, for equal addenda of the wheel and the pinion, the addendum radius of the wheel decides whether the interference will occur or not.

10.14 MINIMUM NUMBER OF TEETH

As explained in the previous section, the maximum value of the addendum radius of the wheel to avoid interference can be up to BE. Referring Fig. 10.24,

$$
\begin{aligned}
(BE)^2 &= (BF)^2 + (FE)^2 \\
&= (BF)^2 + (FP + PE)^2 \\
&= (R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2 \\
&= R^2\cos^2\varphi + R^2\sin^2\varphi + r^2\sin^2\varphi + 2\,r\,R\sin^2\varphi \\
&= R^2(\cos^2\varphi + \sin^2\varphi) + \sin^2\varphi\,(r^2 + 2rR) \\
&= R^2 + (r^2 + 2rR)\sin^2\varphi \\
&= R^2\left[1 + \frac{1}{R^2}(r^2 + 2rR)\sin^2\varphi\right] \\
&= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\varphi\right]
\end{aligned}
$$

$$BE = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi}$$

Therefore, the maximum value of the addendum of the wheel can be equal to (BE - Pitch circle radius) or

$$
\begin{aligned}
a_{w\max} &= R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - R \\
&= R\left[\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - 1\right]
\end{aligned}
$$

Let t = number of teeth on the pinion

T = number of teeth on the wheel

Now, $R = \frac{mT}{2}, r = \frac{mt}{2}$ and $G = \frac{T}{t}$ = Gear ratio

Hence, $$a_{w\max} = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi} - 1\right] = \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right]$$

Let the adopted value of the addendum in some case be a_w times the module of teeth. Then this adopted value of the addendum must be less than the maximum value of the addendum to avoid interference.

i.e.

$$\frac{mT}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1\right] \geq a_w m$$

or

$$T \geq \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1}$$

In the limit,

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1} \qquad (10.8)$$

This gives the minimum number of teeth on the wheel for the given values of the gear ratio, the pressure angle and the *addendum coefficient* a_w.

The minimum number of teeth on the pinion is given by,

$$t = \frac{T}{G}$$

For $a_w = 1$, i.e., when the addendum is equal to one module,

$$T \geq \frac{2}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1}$$

For equal number of teeth on the pinion and the wheel, $G = 1$ and

$$T_{min} = \frac{2}{\sqrt{1+3\sin^2\varphi}-1}$$

For a pressure angle of 20°, i.e., $\varphi = 20°$

$$T_{min} = \frac{2}{\sqrt{1+3\sin^2 20°}-1} = 12.31 \text{ or } 13$$

Thus, for two wheels of equal size with 20° pressure angle and addendum equal to one module, the minimum number of teeth on each wheel must be 13 to avoid interference.

- In case of pinion, the maximum value of the addendum radius to avoid interference is AF (Fig. 10.24) and thus

  $(AF)^2 = (r\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$

  and it can be shown that maximum value of the addendum of the pinion is

$$a_{p\,max} = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\varphi}-r = \frac{mt}{2}\left[\sqrt{1+G(G+2)\sin^2\varphi}-1\right]$$

Example 10.8 *Two 20° involute spur gears mesh externally and give a velocity ratio of 3. The module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine the*

(i) minimum number of teeth on each wheel to avoid interference

(ii) contact ratio

Solution $\varphi = 20°$ $N_p = 120$ rpm

$VR = 3$ Addendum = 1.1 m

$m = 3$ mm $\alpha_w = 1.1$

(i) $$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$

$$= \frac{2\times 1.1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°} - 1} = 49.44$$

Taking the higher whole number divisible by the velocity ratio,

i.e., $T = 51$ and $t = \frac{51}{3} = 17$

(ii) Contact ratio or number of pairs of teeth in contact,

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

$$= \left(\frac{\text{Path of contact}}{\cos\varphi}\right)\times\frac{1}{\pi m}$$

or

$$n = \frac{\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi + \sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi}{\cos\varphi\times\pi m}$$

We have, $R = \frac{mT}{2} = \frac{3\times 51}{2} = 76.5$ mm

$R_a = R + 1.1\,m = 76.5 + 1.1\times 3 = 79.8$ mm

$r = \frac{mt}{2} = \frac{3\times 17}{2} = 25.5$ mm

$r_a = 25.5 + 1.1\times 3 = 28.8$ mm

$$n = \frac{\left[\begin{array}{l}\sqrt{(79.8)^2 - (76.5\cos 20°)^2} - 76.5\sin 20° \\ +\sqrt{(28.8)^2 - (25.5\cos 20°)^2} - 25.5\sin 20°\end{array}\right]}{\cos 20°\times\pi\times 3}$$

$$= \frac{34.646 - 26.165 + 15.977 - 8.720}{\cos 20°\times\pi\times 3}$$

$= 1.78$

Thus, 1 pair of teeth will always remain in contact whereas for 78% of the time, 2 pairs of teeth will be in contact.

Example 10.9 *Two involute gears in a mesh have a velocity ratio of 3. The arc of approach is not to be less than the circular pitch when the pinion is the driver. The pressure angle of the involute teeth is 20°. Determine the least number of teeth on each gear. Also, find the addendum of the wheel in terms of module.*

Solution $\varphi = 20°$; $VR = 3$;

Arc of approach = Circular pitch = πm .... (Given)

$\therefore$ Path of approach = $\pi m\cos 20° = 2.952\,m$

Maximum length of path of approach

$= r\sin\phi = \frac{mt}{2}.\sin 20° = 0.171\,mt$

$\therefore$ $0.171\,mt = 2.952\,m$ or $t = 17.26$ say 18 teeth

and $T = 18\times 3 = 54$

Maximum addendum of the wheel,

$$a_{w\max} = \frac{mT}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1\right]$$

$$= \frac{m\times 54}{2}\left[\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°} - 1\right] = 1.2\,m$$

Example 10.10 *Two 20° involute spur gears have a module of 10 mm. The addendum is equal to one module. The larger gear has 40 teeth while the pinion has 20 teeth.*

Will the gear interfere with the pinion?

Solution $\varphi = 20°$; $T = 40$; $t = 20$; $m = 10$ mm;

Addendum = 1 m = 10 mm

$R = \frac{mT}{2} = \frac{10\times 40}{2} = 200$ mm;

$R_a = 200 + 10 = 210$ mm

$r = \frac{mt}{2} = \frac{10\times 20}{2} = 100$ mm;

$r_a = 100 + 10 = 110$ mm

Let pinion be the driver (Refer Fig.10.24),

Path of approach,

$$PC = \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\phi$$

$= \sqrt{(210)^2 - (200 \times \cos 20°)^2} - 200 \sin 20°$

$= 25.3$ mm

To avoid interference, the maximum length of the path of approach can be PE.

$PE = r \sin \varphi = 100 \sin 20° = 34.2$ mm

Since the actual path of approach is within the maximum limit, no interference occurs.

Alternative Method Addendum radius of wheel = 210 mm

To avoid interference, maximum addendum radius $R_{a\,max}$ can be equal to BE.

i.e., $R_{a\,max} = BE = \sqrt{(BF)^2 + (FP + PE)^2}$

$= \sqrt{(R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2}$

$= \sqrt{(200\cos 20°)^2 + (200\sin 20° + 100\sin 20°)^2}$

$= 214.1$ mm

As the actual addendum radius of the wheel is lesser than the maximum permissible value of the addendum radius, no interference occurs.

Example 10.11 *Two 20° involute spur gears have a module of 10 mm. The addendum is one module. The larger gear has 50 teeth and the pinion has 13 teeth. Does interference occur? If it occurs, to what value should the pressure angle be changed to eliminate interference?*

Solution $\varphi = 20°$; $T = 50$; $m = 10$ mm; $t = 13$;

Addendum = 1 m = 10 mm

$R = \dfrac{mT}{2} = \dfrac{10 \times 50}{2} = 250$ mm;

$R_a = 250 + 10 = 260$ mm

$r = \dfrac{mt}{2} = \dfrac{10 \times 13}{2} = 65$ mm

Refer Fig.10.24,

$R_{a\,max} = \sqrt{(R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2}$

$= \sqrt{(250\cos 20°)^2 + (250\sin 20° + 65\sin 20°)^2}$

$= \sqrt{(250\cos 20°)^2 + (315\sin 20°)^2}$

$= 258.45$ mm

The actual addendum radius R_a is more than the maximum value $R_{a\,max}$, and therefore, interference occurs.

- Maximum addendum radius can also be found using the relation

$$R_{a\,max} = R\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi}$$

$$= 250\sqrt{1 + \frac{13}{50}\left(\frac{13}{50} + 2\right)\sin^2\varphi} = 258.45 \text{ mm}$$

The new value of φ can be found by taking $R_{a\,max}$ equal to R_a.

i.e., $260 = \sqrt{(250\cos\varphi)^2 + (315\sin\varphi)^2}$

or $(260)^2 = (250)^2\cos^2\varphi + (315)^2(1 - \cos^2\varphi)$

$= (250)^2\cos^2\varphi + (315)^2 - (315)^2\cos^2\varphi$

or $\cos^2\varphi = \dfrac{(315)^2 - (260)^2}{(315)^2 - (250)^2} = 0.861$

$\cos\varphi = 0.928$ or $\varphi = 21.88°$ or $21°52'$

Thus, if the pressure angle is increased to $21°52'$, the interference is avoided.

Example 10.12 *The following data relate to two meshing involute gears:*

Number of teeth on the gear wheel	=	*60*
Pressure angle	=	*20°*
Gear ratio	=	*1.5*
Speed of the gear wheel	=	*100 rpm*
Module	=	*8 mm*

The addendum on each wheel is such that the path of approach and the path of recess on each side are 40% of the maximum possible length each. Determine the addendum for the pinion and the gear and the length of the arc of contact.

Solution $R = \dfrac{mT}{2} = \dfrac{8 \times 60}{2} = 240$ mm;

$r = \dfrac{mT}{2} = \dfrac{8 \times (60/1.5)}{2} = 160$ mm

Refer Fig. 10.24 and let the pinion be the driver.

Maximum possible length of path of approach = $r\sin\varphi$

Actual length of path of approach = 0.4 × r sin φ

Similarly, actual length of path of recess = 0.4 R sin φ

Thus, we have

$$0.4r\sin\varphi = \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi$$

$$0.4\times160\ \sin 20^\circ = \sqrt{R_a^2 - (240\cos 20^\circ)^2} - 240\sin 20^\circ$$

$$R_a^2 - 50862 = 10809.8$$

$$R_a^2 = 61671.8$$

$$R_a = 248.3 \text{ mm}$$

Addendum of the wheel = 248.3 − 240 = 8.3 mm

Also, $0.4R\ \sin\varphi = \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$

$$0.4\times240\ \sin 20^\circ = \sqrt{r_a^2 - (160\cos 20^\circ)^2} - 160\sin 20^\circ$$

or $r_a^2 - 22\,605 = 7666$

or $r_a^2 = 30\,271$

or $r_a = 174$ mm

Addendum of the pinion = 174 − 160 = 14 mm

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

$$= 0.4\left(\frac{r\sin\varphi + R\sin\varphi}{\cos\varphi}\right)$$

$$= 0.4\times(240+160)\frac{\sin 20^\circ}{\cos 20^\circ} = 58.2 \text{ mm}$$

Example 10.13 *A pinion of 20° involute teeth rotating at 275 rpm meshes with a gear and provides a gear ratio of 1.8. The number of teeth on the pinion is 20 and the module is 8 mm. If the interference is just avoided, determine (i) the addenda on the wheel and the pinion (ii) the path of contact, and (iii) the maximum velocity of sliding on both sides of the pitch point.*

Solution $\varphi = 20^\circ$; $VR = 1.8$; $m = 8$ mm; $t = 20$; $G = 1.8$; $T = 20\times1.8 = 36$; $N = 275$ rpm

$$R = \frac{mT}{2} = \frac{8\times36}{2} = 144 \text{ mm};\ r = \frac{144}{1.8} = 80 \text{ mm}$$

Maximum addendum of the wheel,

$$a_{w\max} = R\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1\right]$$

$$= 144\left[\sqrt{1+\frac{1}{1.8}\left(\frac{1}{1.8}+2\right)\sin^2 20^\circ} - 1\right]$$

$$= 144\,(1.08 - 1) = 11.5 \text{ mm}$$

Maximum addendum of the pinion,

$$a_{p\max} = r\left[\sqrt{1+G(G+2)\sin^2\varphi} - 1\right]$$

$$= 80\left[\sqrt{1+1.8(1.8+2)\sin^2 20^\circ} - 1\right] = 27.34 \text{ mm}$$

Path of contact when the interference is just avoided

= maximum length of path of approach + maximum length of path of recess

= r sin φ + R sin φ = 80 sin 20° + 144 sin 20°

= 27.36 + 49.24 = 76.6 mm

$$\omega_p = \frac{2\pi\times275}{60} = 28.8 \text{ rad/s};\ \omega_g = \frac{28.8}{1.8} = 16 \text{ rad/s}$$

Velocity of sliding on one side = $(\omega_p + \omega_g)$ × Path of approach

= (28.8 + 16) × 27.36 = 1226 mm/s or 1.226 m/s

Velocity of sliding on other side = $(\omega_p + \omega_g)$ × Path of recess

= (28.8 + 16) × 49.24 = 2206 mm/s or 2.206 m/s

Example 10.14 *The centre distance between two spur gears in a mesh is to be approximately 275 mm. The gear ratio is 10 to 1. The pinion transmits 360 kW at 1800 rpm. The pressure angle of the involute teeth is 20° and the addendum is equal to one module. The limiting value of normal tooth pressure is 1kN/mm of width. Determine the*

(i) nearest standard module so that interference does not occur,

(ii) number of teeth on each gear wheel, and

(iii) width of pinion.

Solution $\varphi = 20°$; $VR = 10$; $C = 275$ mm; $P = 360$ kW; $p = 1$ kN/mm of width; $N_p = 1800$ rpm

$$VR = \frac{N_p}{N_g} = \frac{T_g}{T_p} = \frac{d_g}{d_p} \text{ or } d_g = 10\, d_p$$

$$C = \frac{d_p + d_g}{2} \text{ or } d_p + d_g = 2 \times 275 = 550 \text{ mm}$$

or $11 d_p = 550$ or $d_p = 50$ mm and
$d_g = 50 \times 10 = 500$ mm

Minimum number of teeth on gear wheel,

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$$

$$= \frac{2 \times 1}{\sqrt{1 + \frac{1}{10}\left(\frac{1}{10} + 2\right)\sin^2 20°} - 1} = 164$$

(i) Number of teeth on pinion = 164/10 = 16.4 say 17

∴ number of teeth on gear wheel
= 17 × 10 = 170

(ii) Now $m = \frac{d_p}{t} = \frac{50}{17} \approx 3$ mm

Exact $d_p = m\, T_p = 3 \times 17 = 51$ mm and
$d_g = m\, T_g = 3 \times 170 = 510$ mm

Exact centre distance,

$$C = \frac{d_p + d_g}{2} = \frac{51 + 510}{2} = 280.5 \text{ mm}$$

(iii) $P = \frac{2\pi NT}{60}$ or $360 \times 1000 = \frac{2\pi \times 1800 \times T}{60}$

or $T = 1909.9$ N.m

$$\text{Tangential force} = \frac{1909.9 \times 10^3}{51/2} = 74\,896 \text{ N}$$

Normal pressure on the tooth

$$= \frac{F}{\cos\varphi} = \frac{74\,896}{\cos 20°} = 79\,700 \text{ N}$$

$$\text{Width of pinion} = \frac{F_n}{\text{Limiting normal pressure}}$$

$$= \frac{79\,700}{1000} = 79.7 \text{ mm}$$

10.15 INTERFERENCE BETWEEN RACK AND PINION

Figure 10.27 shows a rack and pinion in which pinion is rotating in the clockwise direction and driving the rack. P is the pitch point and PE is the line of action. Engagement of the rack tooth with the pinion tooth occurs at C. To avoid interference, the maximum addendum of the rack can be increased in such a way that C coincides with E. Thus, the addendum of the rack must be less than GE.

Let the adopted value of the addendum of the rack be $a_r m$ where a_r is the *addendum coefficient* by which the standard value of the addendum has been multiplied.

$$GE = PE \sin\phi = (r \sin\phi)\sin\varphi = r\sin^2\phi$$

$$= \frac{mt}{2}\sin^2\varphi$$

To avoid interference,

$$GE \geq a_r m \text{ or } \frac{mt}{2}\sin^2\varphi \geq a_r m \text{ or } t \geq \frac{2a_r}{\sin^2\varphi}$$

When $a_r = 1$, i.e., for standard addendum,

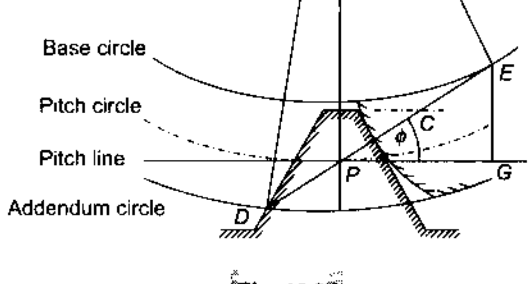


Fig. 10.27

$$t_{min} \geq \frac{2}{\sin^2 \varphi}$$

For 20° pressure angle, $\phi = 20°$, $\therefore$ $t_{min} = 17.1$ or 18

Thus, the number of minimum teeth on the pinion to avoid interference is 18.

$$\text{Path of contact} = CP + DP = \frac{\text{Add. of rack}}{\cos \varphi} + \sqrt{r_a^2 - (r \cos \varphi)^2} - r \sin \varphi$$

$$\text{Maximum path of contact to avoid interference} = DE = \sqrt{r_a^2 - (r \cos \varphi)^2}$$

Example 10.15 *A pinion of 32 involute teeth and 4 mm module drives a rack. The pressure angle is 20°. The addendum of both pinion and rack is the same. Determine the maximum permissible value of the addendum to avoid interference. Also, find the number of pairs of teeth in contact.*

Solution $t = 32$; $m = 4$ mm; $\phi = 20°$;

$$r = \frac{mt}{2} = \frac{4 \times 32}{2} = 64 \text{ mm}$$

Refer Fig. 10.27,

To avoid interference, the maximum value of addendum = GE

$= r \sin^2 \varphi = 64 \sin^2 20° = \underline{7.487 \text{ mm}}$

Addendum radius of the pinion

$= 64 + 7.487 = 71.487$ mm

Maximum path of contact to avoid interference = DE

$$= \sqrt{r_a^2 - (r \cos \varphi)^2} = \sqrt{71.487^2 - (64 \cos 20°)^2}$$

$= 38.646$ mm

Number of pairs of teeth in contact

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \left(\frac{\text{Path of contact}}{\cos \varphi}\right) \times \frac{1}{\pi m}$$

$$= \left(\frac{38.646}{\cos 20°}\right) \times \frac{1}{\pi \times 4} = 3.27$$

Thus, 3 pairs of teeth will always remain in contact whereas for 27% of the time, 4 pairs of teeth will be in contact.

10.16 UNDERCUTTING

Figure 10.28(a) shows a pinion. A portion of its dedendum falls inside the base circle. The profile of the tooth inside the base circle is radial. If the addendum of the mating gear is more than the limiting value, it interferes with the dedendum of the pinion and the two gears are locked.

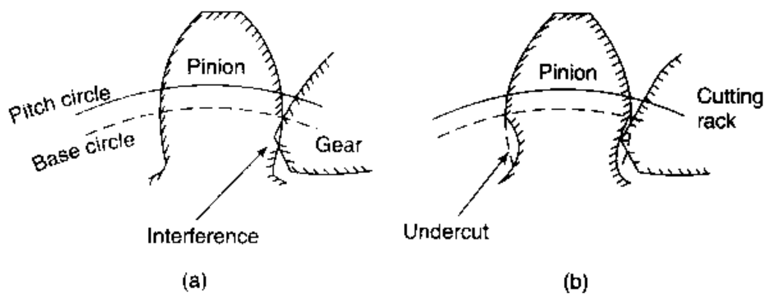


Fig. 10.28

However, if a cutting rack having similar teeth is used to cut the teeth in the pinion, it will remove that portion of the pinion tooth which would have interfered with the gear as shown in Fig. 8.28(b). A gear having its material removed in this manner is said to be *undercut* and the process, *undercutting*. In a pinion with small number of teeth, this can seriously weaken the tooth. However, when the actual gear meshes with the undercut pinion, no interference occurs.

Undercutting will not take place if the teeth are designed to avoid interference.

10.17 COMPARISON OF CYCLOIDAL AND INVOLUTE TOOTH FORMS

The following table compares the two forms of teeth, the cycloidal and the involute:

Table 10.2 Comparison of cycloidal and involute teeth

	Cycloidal Teeth	Involute Teeth
(a)	Pressure angle varies from maximum at the beginning of engagement, reduces to zero at the pitch point and again increases to maximum at the end of engagement resulting in less smooth running of the gears.	Pressure angle is constant throughout the engagement of teeth. This results in smooth running of the gears.
(b)	It involves double curve for the teeth, epicycloid and hypocycloid. This complicates the manufacture.	It involves single curve for the teeth resulting in simplicity of manufacturing and of tools.
(c)	Owing to difficulty of manufacture, these are costlier.	These are simple to manufacture and thus are cheaper.
(d)	Exact centre-distance is required to transmit a constant velocity ratio.	A little variation in the centre distance does not affect the velocity ratio.
(e)	Phenomenon of interference does not occur at all.	Interference can occur if the condition of minimum number of teeth on a gear is not followed.
(f)	The teeth have spreading flanks and thus are stronger.	The teeth have radial flanks and thus are weaker as compared to the cycloidal form for the same pitch.
(g)	In this, a convex flank always has contact with a concave face resulting in less wear.	Two convex surfaces are in contact and thus there is more wear.

On careful examination of the above, it can be deduced that the advantages of involute system are more real. Therefore, the use of involute teeth have become almost universal rendering the cyclodial system obsolete.

10.18 HELICAL AND SPIRAL GEARS

In helical and spiral gears, the teeth are inclined to the axis of a gear. They can be *right-handed* or *left-handed*, depending upon the direction in which the helix slopes away from the viewer when a gear is viewed parallel to the axis of the gear. In Fig. 10.29, the gear 1 is a right-handed helical gear whereas 2 is left-handed. The two mating gears have parallel axes and equal helix angle ψ. The contact between two teeth on the two gears is first made at one end which extends through the width of the wheel with the rotation of the gears.

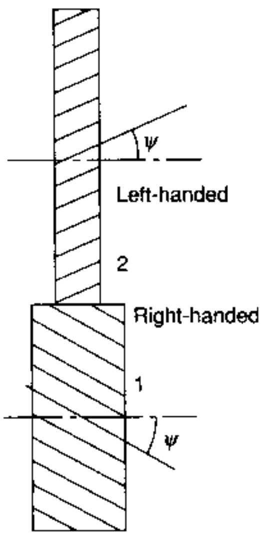


Fig. 10.29

Figure 10.30(a) shows the same two gears when looking from above. Now, if the helix angle of the gear 2 is reduced by a few degrees so that the helix angle of the gear 1 is ψ_1 and that of gear 2 is ψ_2 and it is desired that the teeth of the two gears still mesh with each other tangentially, it is essential to rotate the axis of the gear 2 through some angle as shown in Fig. 10.30(b). Let the angle turned by it be θ which is the angle between the axes of the two gears.

From the geometry of the diagram, $\theta = \psi_1 - \psi_2$.

Thus, this is the case of two skew shafts joined with crossed-helical or spiral gears.

When $\psi_1 = \psi_2$, the helix angle is the same as before. Then $\theta = \psi_1 - \psi_2 = 0$, a case of helical gears joining parallel shafts [Fig. 10.30(a)].

When $\psi_2 = 0$, i.e., the helix angle of the gear 2 is made zero, or the gear 2 is a straight spur gear, then $\theta = \psi_1$, i.e., the angle between the axes becomes equal to the helix angle of the gear 1 [Fig. 10.30(c)],

In case the helix angle of gear 2 is made negative, i.e. the teeth are made of the same hand as that of gear 1 (Fig. 10.30(d)]

$\theta = \psi_1 - (-\psi_2) = \psi_1 + \psi_2$

The above discussion leads to the following conclusion:

Angle between the shafts,

$\theta = \psi_1 + \psi_2$ for gears of same hand (10.9)

$= \psi_1 - \psi_2$ for gears of opposite hands (10.10)

In case of helical gears for parallel shafts there is a line contact of the teeth through the width. However, gears for skew shafts have a point contact. This can be demonstrated by having two cylinders. When their axes are parallel, they can have a line contact. But when any of the cylinders is turned through some angle so that their axes are no longer parallel, the contact is reduced to a point. Thus, whereas the helical gears for parallel shafts are considered stronger than the spur gears, the crossed-helical gears (spiral gears) for skew shafts are not used to transmit heavy loads.

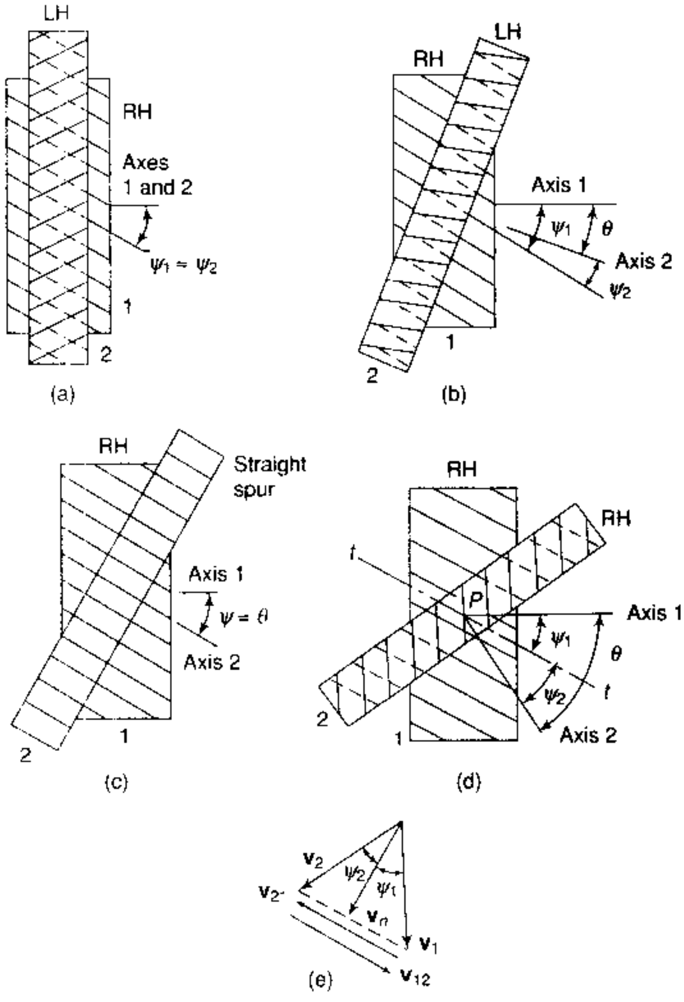


Fig. 10.30

The pitch line velocities v_1 and v_2 of the gears 1 and 2 of Fig. 10.30(d) act in the directions as shown in Fig 10.30(e). The magnitude and direction of v_{12} represent the sliding velocity of the teeth of the gear 1 relative to that of the gear 2 parallel to $t - t$, the tangent to the teeth in contact. This velocity increases with the increase in the angle between the shafts and is not zero even when the contact is at the pitch point.

However, in all cases, the normal components of v_1 and v_2, i.e., the components perpendicular to $t - t$ must be equal.

For helical gears joining parallel shafts, $v_1 = v_2$ in the same direction.

$v_{12} = 0$ or there is no sliding velocity.

If the helix angle of a gear is increased, a gear similar to that shown in Figure 10.31 is obtained. This gives the appearance of a spiral and thus the name.

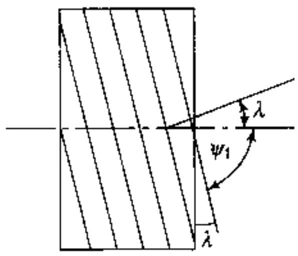


Fig. 10.31

10.19 TERMINOLOGY OF HELICAL GEARS

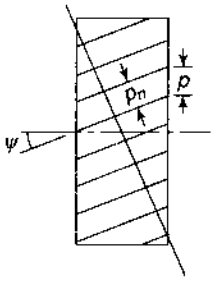


Fig. 10.32

Refer Fig. 10.32.

Helix Angle (ψ) It is the angle at which the teeth are inclined to the axis of a gear. It is also known as *spiral angle*.

Circular Pitch (p) It is the distance between the corresponding points on adjacent teeth measured on the pitch circle. It is also known as *transverse circular pitch*.

Normal Circular Pitch (p_n) Normal circular pitch or simply normal pitch is the shortest distance measured along the normal to the helix between corresponding points on the adjacent teeth. The normal circular pitch of two mating gears must be same.

$$P_n = p \cos \psi$$

Also, we have, $p = \pi m$ as for spur gears

$$P_n = \pi m_n$$

and $$m_n = m \cos \psi$$

10.20 VELOCITY RATIO AND CENTRE DISTANCE OF HELICAL GEARS

Velocity Ratio Refer Fig. 10.30(e).

$$v_n = v_1 \cos \psi = v_2 \cos \psi_2$$

or $$\frac{v_2}{v_1} = \frac{\cos \psi_1}{\cos \psi_2}$$

$$VR = \frac{\omega_2}{\omega_1} = \frac{v_2 / r_2}{v_1 / r_1} = \frac{v_2 / d_2}{v_1 / d_1} = \frac{d_1 / v_2}{d_2 / v_1}$$

or $$VR = \frac{d_1}{d_2} \frac{\cos \psi_1}{\cos \psi_2}$$

$$= \frac{m_1 T_1}{m_2 T_2} \frac{\cos \psi_1}{\cos \psi_2}$$

$$= \frac{m_n / \cos \psi_1}{m_n / \cos \psi_2} \frac{T_1}{T_2} \frac{\cos \psi_1}{\cos \psi_2}$$

$$= \frac{T_1}{T_2} \qquad (10.11)$$

Centre Distance Let C be the centre distance between two skew shaft axes which is the shortest distance between them.

$$C = r_1 + r_2 = \frac{1}{2}(d_1 + d_1) = \frac{1}{2}(m_1 T_1 + m_2 T_2)$$

or $$C = \frac{1}{2}\left(\frac{m_n}{\cos \psi_1} \cdot T_1 + \frac{m_n}{\cos \psi_2} T_2 \right)$$

$$= \frac{m_n}{2}\left(\frac{T_1}{\cos\psi_1}+\frac{T_2}{\cos\psi_2}\right)$$

In case of helical gears for parallel shafts, $\psi_1 = \psi_2 = \psi$

$$C = \frac{m_n}{2\cos\psi}(T_1 + T_2) \qquad (10.12)$$

10.21 HELICAL GEAR FORCES AND EFFICIENCY

Like all direct contact mechanisms, the force exerted by a helical gear on its mating gear acts normal to the contacting surfaces if friction is neglected. However, a normal force in case of helical gears has three components. Apart from tangential and radial components which are present in spur gears, a third component parallel to the axis of the shaft of the gear also exists. This is known as the axial or the thrust force component. Figure 10.33 shows the normal force and its components acting on a helical gear. The gear shown is the driven gear and the forces are exerted on it by the driving gear.

Let F_n' = total normal force

F_t = tangential force

F_a = axial force

F_r = radial force

F_n = normal force in the plane of F_t and F_a

φ = pressure angle

φ_n = normal pressure angle

ψ = helix angle

Then

$F_n = F_n' \cos\varphi_n$ and $F_r = F_n' \sin\varphi_n$

$F_t = F_n \cos\psi$ and $F_a = F_n \sin\psi$

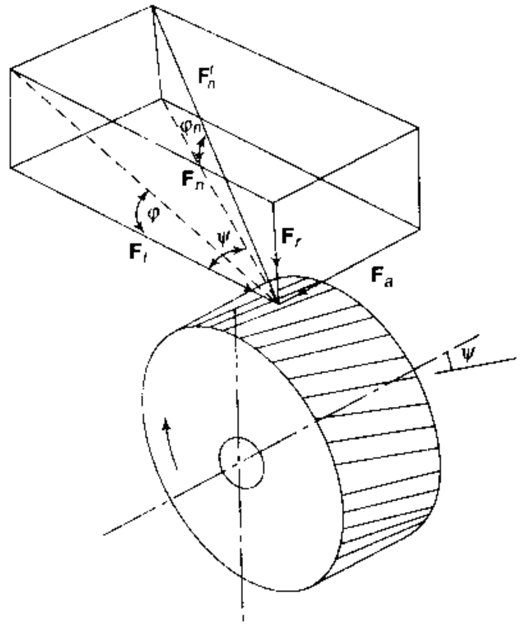


Fig. 10.33

Efficiency of Spiral and Helical Gears

In spiral or crossed helical gears, the sliding action between the surfaces acts chiefly along the tangent to the pitch helix. The friction force is equal to μF_n and acts in a direction opposite to the direction of sliding of the gear surface.

Two mating spiral gears 1 and 2 are shown separately in Figs 10.34(a) and (b) along with the force acting on them. Gear 1 is driving the gear 2.

Let F_{t1} = tangential force acting on the gear wheel 1

F_{t2} = tangential force acting on the gear wheel 2

F_{n1} = $F_{n2} = F_n$ normal reaction force between two surfaces in contact

The direction of the sliding velocity of the gear 2 relative to that of the gear 1, v_{21} has been shown in Fig. 10.34(c). The friction force $\mu\mathbf{F}_{n2}$ acts in the opposite direction. $\mathbf{F}_{n2}$ and $\mu\mathbf{F}_{n2}$ combine into one reaction force $\mathbf{F}_2$ inclined at an angle φ with the normal reaction, where φ is the angle of friction.

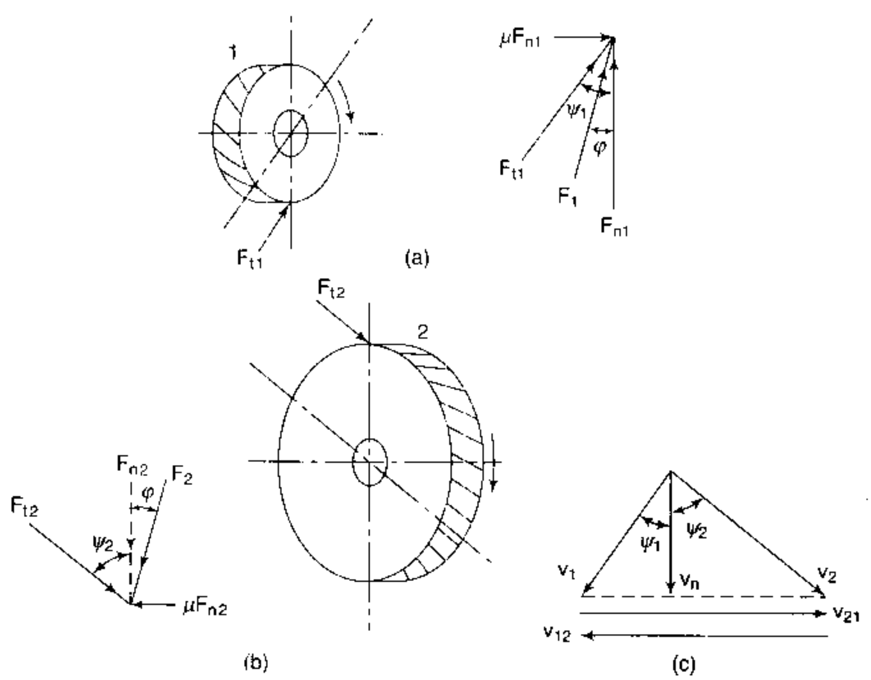


Fig. 10.34

$F_{t2} = F_2 \cos(\psi_2 + \varphi)$ and $F_{t1} = F_1 \cos(\psi_1 - \varphi)$

But as $F_n = F_{n1} = F_{n2}$ and $\mu F_{n1} = \mu F_{n2}$

$F_1 = F_2 = F$

or $F_{t1} = F \cos(\psi_1 - \varphi)$ and $F_{t2} = F \cos(\psi_2 + \varphi)$

Input = $F_{t1} \times v_1$

output = $F_{t2} \times v_2$

Efficiency, $\eta = \dfrac{F_{t2} \times v_2}{F_{t1} \times v_1}$

$$= \frac{\cos(\psi_2 + \varphi)}{\cos(\psi_1 - \varphi)} \frac{\cos\psi_1}{\cos\psi_2} \qquad \left(\frac{v_2}{v_1} = \frac{\cos\psi_1}{\cos\psi_2}\right) \tag{10.13}$$

$$= \frac{2\cos\psi_1 \cos(\psi_2 + \varphi)}{2\cos\psi_2 \cos(\psi_1 - \varphi)}$$

$$= \frac{\cos(\psi_1 + \psi_2 + \varphi) + \cos(\psi_1 - \psi_2 - \varphi)}{\cos(\psi_2 + \psi_1 - \varphi) + \cos(\psi_2 - \psi_1 + \varphi)}$$

$$= \frac{\cos(\theta + \varphi) + \cos(\psi_1 - \psi_2 - \varphi)}{\cos(\theta - \varphi) + \cos[-(\psi_1 - \psi_2 - \varphi)]}$$

$$= \frac{\cos(\theta + \varphi) + \cos(\psi_1 - \psi_2 - \varphi)}{\cos(\theta - \varphi) + \cos(\psi_1 - \psi_2 - \varphi)} \qquad [\text{As } \cos(-\alpha) = \cos\alpha]$$

The numerator and the denominator, each is a sum of two terms, out of which one is common. The expression will be maximum if the common term is maximum.

i.e., $\cos(\psi_1 - \psi_2 - \varphi)$ is maximum or equal to 1.

or $\quad \psi_1 - \psi_2 - \varphi = 0$

or $\quad \psi_1 = \psi_2 + \varphi = (\theta - \psi_1) + \varphi \qquad (\theta = \psi_1 + \psi_2)$

or $\quad 2\psi_1 = \theta + \varphi \qquad$ or $\qquad \psi_1 = \dfrac{\theta + \varphi}{2}$

and
$$\eta_{max} = \frac{\cos(\theta + \varphi) + 1}{\cos(\theta - \varphi) + 1} \tag{10.14}$$

Example 10.16 *Two spiral gears have a normal module of 12 mm and the angle between the shaft axes is 60°. The driver has 16 teeth and a helix angle of 25°. If the velocity ratio is 1/2 and the driver and the follower both are left-handed, find the centre distance between the shafts.*

Solution $\psi_1 = 25°$; $m_n = 12$ mm;

$\psi_2 = 60° - 25° = 35°$;

$T_1 = 16$

$$VR = \frac{1}{2} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$T_2 = \frac{T_1}{VR} = \frac{16}{1/2} = 32$$

$$C = \frac{m_n}{2}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right)$$

$$= \frac{12}{2}\left(\frac{16}{\cos 25°} + \frac{32}{\cos 35°}\right)$$

or $\quad C = \underline{340.3 \text{ mm}}$

Example 10.17 *The centre distance between two meshing spiral gears is 260 mm and the angle between the shafts is 65°. The normal circular pitch is 14 mm and the gear ratio is 2.5. The driven gear has a helix angle of 35°. Find the*

(i) number of teeth on each wheel

(ii) exact centre distance

(iii) efficiency assuming the friction angle to be 5.5°

Solution $\psi_2 = 35° \qquad G = 2.5$

$\psi_1 = 65° - 35° = 30° \qquad C = 260$ mm

$p_n = 14$ mm $\qquad \varphi = 5.5°$

Let the gear with smaller number of teeth be the driver.

$$G = \frac{T_2}{T_1} = 2.5 \text{ or } T_2 = 2.5T_1$$

$$C = \frac{p_n}{2\pi}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right) \qquad \left(m_n = \frac{p_n}{\pi}\right)$$

$$260 = \frac{14}{2\pi}\left(\frac{T_1}{\cos 30°} + \frac{2.5T_1}{\cos 35°}\right) = 9.373\,T_1$$

or $\quad T_1 = 27.74$

Take $T_2 = \underline{28}$

Then $T_2 = 2.5 \times 28 = \underline{70}$

$$C_{exact} = \frac{14}{2\pi}\left(\frac{28}{\cos 30°} + \frac{70}{\cos 35°}\right)$$

$= \underline{262.4 \text{ mm}}$

$$\eta = \frac{\cos(\psi_2 + \varphi)\cos\psi_1}{\cos(\psi_1 - \varphi)\cos\psi_2}$$

$$= \frac{\cos(35° + 5.5°)\cos 30°}{\cos(30° - 5.5°)\cos 35°}$$

$= \underline{0.883}$

Example 10.18 *Two left-handed helical gears connect two shafts 60° apart. The normal module is 6 mm. The larger gear has 70 teeth and the velocity ratio is 1/2. The centre distance is 370 mm. Find the helix angles of the two gears.*

Solution $\theta = 60°$ $m_n = 6$ mm
$T_2 = 70$ $C = 370$ mm
$\psi_2 = 60° - \psi_1$

$$VR = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

or $$T_1 = VR \times T_2 = \frac{1}{2} \times 70 = 35$$

$$C = \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos(60° - \psi_1)}\right]$$

$$370 = \frac{6}{2}\left[\frac{35}{\cos\psi_1} + \frac{70}{\cos(60° - \psi_1)}\right]$$

By trial and error, $\psi_1 = 26°$, $\psi_2 = 34°$

Example 10.19 *The following data relate to two spiral gears in mesh:*

Shaft angle = 90°
Centre distance = 160 mm (approx)
Normal circular pitch = 8 mm
Gear ratio = 3
Friction angle = 5°

For maximum efficiency of the drive, determine the

(i) spiral angles of the teeth
(ii) number of teeth
(iii) centre distance (exact)
(iv) pitch diameters
(v) efficiency

Solution

(i) $$\psi_1 = \frac{\theta + \varphi}{2} \quad \text{For maximum efficiency}$$

$$= \frac{90° + 5°}{2}$$

$$= 47.5°$$

$$\psi_2 = 90° - 47.5° = 42.5°$$

(ii) $$C = \frac{p_n}{2\pi}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right)$$

$$160 = \frac{8}{2\pi}\left(\frac{T_1}{\cos 47.5°} + \frac{3T_1}{\cos 42.5°}\right) = 7.065T_1$$

or $T_1 = 22.65$
Let T_1 to be 23
$T_2 = 3T_1 = 69$

(iii) $$C_{exact} = \frac{8}{2\pi}\left(\frac{23}{\cos 47.5°} + \frac{69}{\cos 42.5°}\right)$$

$$= \underline{162.5 \text{ mm}}$$

(iv) $$d_1 = \frac{P_1 T_1}{\pi} = \frac{p_n}{\cos\psi_1}\frac{T_1}{\pi}$$

$$= \frac{8}{\cos 47.5°} \times \frac{23}{\pi} = \underline{86.7 \text{ mm}}$$

$$d_2 = \frac{8}{\cos 42.5°} \times \frac{69}{\pi} = 238.3 \text{ mm}$$

(v) $$\eta_{max} = \frac{\cos(\theta + \varphi) + 1}{\cos(\theta - \varphi) + 1}$$

$$= \frac{\cos(90° + 5°) + 1}{\cos(90° - 5°) + 1} = \underline{0.84}$$

Example 10.20 *A drive is made up of two spiral gear wheels of the same hand, same diameter and of normal pitch of 14 mm. The centre distance between the axes of the shafts is approximately 150 mm. The speed ratio is 1.6 and the angle between the shafts is 75°. Assuming a friction angle of 6°, determine the*

(i) spiral angle of each wheel
(ii) number of teeth on each wheel
(iii) efficiency of the drive
(iv) maximum efficiency

Solution $\theta = 75°$ $m_n = 14$ mm
$C = 150$ mm $\phi = 6°$
$VR = 1.6$

(i) Let ψ_1 be the spiral angle of the wheel 1. Then, the spiral angle of the wheel 2, $\psi_2 = 75° - \psi_1$

Now, Velocity ratio, $VR = \dfrac{T_1}{T_2} = 1.6$

(Refer Eq. 10.11)

or $T_1 = 1.6T_2$

Also, $$VR = \frac{d_1\cos\psi_1}{d_2\cos\psi_2}$$

$$1.6 = \frac{\cos\psi_1}{\cos\psi_2} \qquad (\text{As } d_1 = d_2)$$

$$\text{or } \cos\psi_1 = 1.6\cos(75^\circ - \psi_1)$$

$$= 1.6(\cos 75^\circ \cos\psi_1 + \sin 75^\circ \sin\psi_1)$$

$$= 1.6(0.2588\cos\psi_1 + 0.9659\sin\psi_1)$$

$$= 0.414\cos\psi_1 + 1.545\sin\psi_1$$

$$0.586\cos\psi_1 = 1.545\sin\psi_1$$

$$\tan\psi_1 = 0.3793$$

$$\psi_1 = \underline{20.8^\circ}$$

$$\text{and} \quad \psi_2 = 75^\circ - 20.8^\circ = \underline{54.2^\circ}$$

(ii) The centre distance,

$$C = \frac{p_n}{2\pi}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right) \qquad \left(m_n = \frac{p_n}{\pi}\right)$$

$$150 = \frac{14}{2\pi}\left(\frac{1.6T_2}{\cos 20.8} + \frac{T_2}{\cos 54.2}\right) = 7.623\,T_2$$

$$T_2 = 19.68 \text{ say } \underline{20}$$

$$T_1 = 20 \times 1.6 = 32$$

$$C_{exac} = \frac{14}{2\pi}\left(\frac{32}{\cos 20.8} + \frac{20}{\cos 54.2}\right)$$

$$= 152.4 \text{ mm}$$

(iii) Efficiency,

$$\eta = \frac{\cos(\psi_2 + \varphi)\cos\psi_1}{\cos(\psi_1 - \varphi)\cos\psi_2}$$

$$= \frac{\cos(54.2^\circ + 6^\circ)\cos 20.8^\circ}{\cos(20.8^\circ - 6^\circ)\cos 54.2^\circ}$$

$$= \underline{0.821}$$

(iv) Maximum efficiency.

$$\eta_{max} = \frac{\cos(\theta + \varphi) + 1}{\cos(\theta - \varphi) + 1} = \frac{\cos(75^\circ + 6^\circ) + 1}{\cos(75^\circ - 6^\circ) + 1}$$

$$= \underline{0.872}$$

10.22 WORM AND WORM GEAR

To accomplish large speed reduction in skew shafts, spiral gears with a small driver and a larger follower are required. Also, the load transmitted through these gears is limited. To transmit a little higher load than with the usual spiral gears, use of worm and worm gears (throated type) can be made [Fig. 10.11(b) and (c)]. Usually, worm and worm gears are used to connect two skew shafts at right angles to each other. The axial length of the worm is increased so that at least one or two threads (teeth) complete the circle on it (Fig. 10.31).

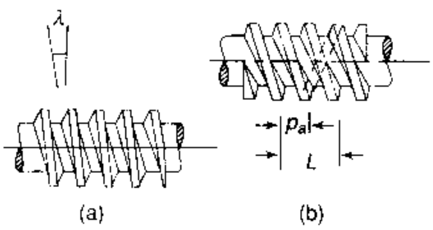
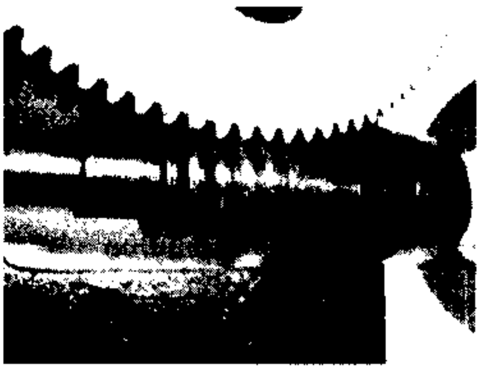


Fig. 10.35

A worm can be a single, double or triple start if one, two or three threads are traversed on the worm for one tooth advancement of the gear wheel. Figures 10.35(a) and (b) show a single start and a double-start worm respectively.



A pair of single-start worm and worm gear

A pair of three-start worm and worm gear

10.23 TERMINOLOGY OF WORM GEARS

Refer Fig. 10.35,

(i) Axial Pitch (p_a) It is the distance between corresponding points on adjacent teeth measured along the direction of the axis.

(ii) Lead (L) The distance by which a helix advances along the axis of the gear for one turn around is known as lead.

In a single helix, the axial pitch is equal to lead. In a double helix, this is one-half the lead, in a triple helix, one third of lead, and so on.

(iii) Lead Angle (λ) It is the angle at which the teeth are inclined to the normal to the axis of rotation. Obviously, the lead angle is the complement of the helix angle.

i.e., $\psi + \lambda = 90°$

In case of worms, the lead angle is very small and the helix angle approaches 90°.

As the shaft axes of worm and worm gear are at 90°,

$$\psi_1 + \psi_2 = 90° \quad \text{(1 denotes worm)}$$

$$(90° - \lambda_1) + \psi_2 = 90°$$

or $$\lambda_1 = \psi_2$$

i.e., lead angle of worm = helix angle of the gear wheel

Also, p_n of worm = p_n of wheel

$$p_{a1}\cos\lambda_1 = p_2\cos\psi_2$$

but $$\lambda_1 = \psi_2$$

$\therefore$ $$p_{a1} = p_2$$

i.e., axial pitch of worm = circular pitch of wheel

10.24 VELOCITY RATIO AND CENTRE DISTANCE OF WORM GEARS

Velocity Ratio As a worm may be multistart, the velocity is not calculated from the number of teeth.

Assume that a worm rotates through one revolution about its axis. Then the angle turned by it will be 2π.

The lead of the worm is equal to the axial distance advanced by a thread in one revolution of the worm.

The lead is also the distance moved by the pitch circle of the gear wheel. Thus, angle turned by it during the same time will be l/R_2 or $2l/d_2$.

$$VR = \frac{\text{Angle turned by the gear}}{\text{Angle turned by the worm}} = \frac{2l/d_2}{2\pi} = \frac{l}{\pi d_2} \quad (10.15)$$

Centre Distance

$$C = \frac{m_n}{2}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right)$$

$$= \frac{m_2\cos\psi_2}{2}\cdot\frac{1}{\cos\psi_2}\left(\frac{\cos\psi_2}{\cos\psi_1}.T_1 + T_2\right)$$

$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\cos(90° - \lambda_1)}T_1 + T_2\right] \quad [\psi_2 = \lambda_1,\ \psi_1 = 90° - \lambda_1]$$

$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\sin\lambda_1}T_1 + T_2\right]$$

$$= \frac{m_2}{2}[T_1 \cot\lambda_1 + T_2] \tag{10.16}$$

10.25 EFFICIENCY OF WORM GEARS

$$\eta = \frac{\cos(\psi_2 + \varphi)\cos\psi_1}{\cos(\psi - \varphi)\cos\psi_2} \qquad \text{[Refer Eq. (10.13)]}$$

$$= \frac{\cos(\lambda_1 + \varphi)}{\cos[90^\circ - \lambda_1 - \varphi]} \cdot \frac{\cos(90^\circ - \lambda_1)}{\cos\lambda_1}$$

$$= \frac{\cos(\lambda_1 + \varphi)}{\cos[90^\circ - (\lambda_1 + \varphi)]} \cdot \frac{\sin\lambda_1}{\cos\lambda}$$

$$= \frac{\cos(\lambda_1 + \varphi)}{\sin(\lambda_1 + \varphi)}\tan\lambda_1$$

$$= \frac{\tan\lambda_1}{\tan(\lambda_1 + \varphi)} \tag{10.17}$$

$$\eta_{max} = \frac{\cos(\theta + \varphi) + 1}{\cos(\theta - \varphi) + 1} = \frac{\cos(90^\circ + \varphi) + 1}{\cos(90^\circ - \varphi) + 1} \qquad (\theta = 90^\circ)$$

$$= \frac{1 - \sin\varphi}{1 + \sin\varphi} \tag{10.17a}$$

If the gear wheel is the driver, it can be deduced that

$$\eta = \frac{\tan(\lambda_1 - \varphi)}{\tan\lambda_1}$$

and

$$\eta_{max} = \frac{1 - \sin\varphi}{1 + \sin\varphi}$$

Example 10.21 *A two-start worm rotating at 800 rpm drives a 26-tooth worm gear. The worm has a pitch diameter of 54 mm and a pitch of 18 mm. If coefficient of frication (μ) is 0.06, find the*

(i) helix angle of worm
(ii) speed of gear
(iii) centre distance
(iv) lead angle for maximum efficiency
(v) efficiency
(vi) maximum efficiency

Solution:

$N_1 = 800$ rpm $\qquad T_2 = 26$

$\mu = \tan\varphi = 0.06 \qquad d_1 = 54$ mm

$\varphi = 3.43^\circ$ (= $\tan^{-1} 0.06$) $\qquad p_1 = 18$ mm

(i) Unwrap one thread of the worm,

$$\tan\lambda_1 = \frac{\text{Lead}}{\text{Pitch circumferece}} = \frac{2p}{\pi d_1}$$

(for two-start worm)

$$\text{or} \quad \tan\lambda_1 = \frac{2 \times 18}{\pi \times 54} = 0.212$$

$$\lambda_1 = 11.98^\circ \text{ or } 11^\circ 59'$$

Helix angle $\psi_1 = 90° - \lambda_1 = \underline{78°01'}$

(ii) Pitch of wheel = Axial pitch of worm

$= 18$ mm

$$\therefore \quad p_2 = \frac{\pi d_2}{T_2}$$

or $$18 = \frac{\pi d_2}{26}$$

or $d_2 = 149$ mm

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{\text{Lead}}{\pi d_2} = \frac{2 \times 18}{\pi \times 149}$$

or $$N_2 = \frac{2 \times 18}{\pi \times 149} \times 800 = \underline{61.5 \text{ rpm}}$$

Alternatively, $$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$N_2 = \frac{2}{26} \times 800 = \underline{61.5 \text{ rpm}}$$

(iii) $$C = \frac{m_2}{2}(T_1 \cot \lambda_1 + T_2) = \frac{p_2}{2\pi}(T_1 \cot \lambda_1 + T_2)$$

$$= \frac{18}{2\pi}(2 \cot 11.98° + 26) = \underline{101.4 \text{ mm}}$$

(iv) For maximum efficiency, $\psi_1 = \dfrac{\theta + \varphi}{2}$

or $$90° - \lambda_1 = \frac{90° + \varphi}{2}$$

or $$\lambda_1 = 45° - \frac{3.43°}{2}$$

$= 43.29°$ or $\underline{43°18'}$

(v) $$\eta = \frac{\tan \lambda_1}{\tan(\lambda_1 + \varphi)}$$

$$= \frac{\tan 11.98°}{\tan(11.98° + 3.43°)} \quad \ldots(\tan^{-1} 0.06 = 3.43°)$$

$= \underline{0.77}$

(vi) $$\eta_{max} = \frac{1 - \sin \varphi}{1 + \sin \varphi} = \frac{1 - \sin 3.43°}{1 + \sin 3.43°} = \underline{0.887}$$

10.26 BEVEL GEARS

To have a gear drive between two intersecting shafts, bevel gears are used. Kinematically, bevel gears are equivalent to rolling cones. Some of the common terms used in bevel gears are illustrated in Fig.10.36(a).

Let γ_g, γ_p = pitch angles of gear and pinion respectively

r_g, r_p = pitch radii of gear and pinion respectively.

The pitch cones for two mating external bevel gears are shown in Fig.10.36(b).

We have,

$$\sin \gamma_g = \frac{r_g}{OP} = \frac{r_g}{r_p / \sin \gamma_p} = \frac{r_g}{r_p} \sin(\theta - \gamma_g)$$

or $$\sin \gamma_g = \frac{r_g}{r_p}(\sin \theta \cos \gamma_g - \cos \theta \sin \gamma_g)$$

Dividing both sides by $\cos \gamma_g$,

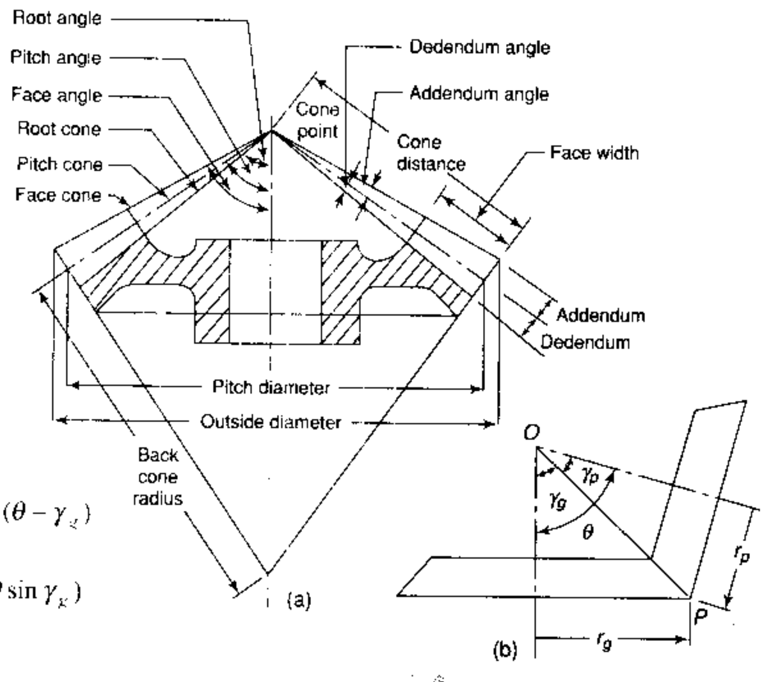
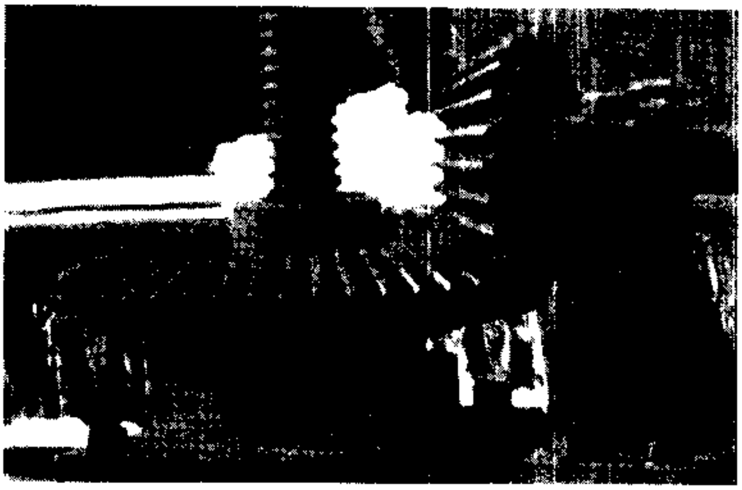


Fig. 10.36



A pair of straight bevel gears

Gear mechanism of a drilling machine showing the use of a pair of bevel gears

$$\tan\gamma_g = \frac{r_g}{r_p}(\sin\theta - \cos\theta\tan\gamma_g)$$

or

$$\frac{r_p}{r_g}\tan\gamma_g = \sin\theta - \cos\theta\tan\gamma_g$$

$$\tan\gamma_g = \frac{\sin\theta}{\dfrac{r_p}{r_g} + \cos\theta}$$

As

$$v_p = \omega_g r_g = \omega_p r_p \quad \text{or} \quad \frac{r_p}{r_g} = \frac{\omega_g}{\omega_p}$$

$\therefore$

$$\tan\gamma_g = \frac{\sin\theta}{\dfrac{\omega_g}{\omega_p} + \cos\theta} \tag{10.18}$$

Similarly,

$$\tan\gamma_p = \frac{\sin\theta}{\dfrac{\omega_p}{\omega_g} + \cos\theta} \tag{10.19}$$

Example 10.22 *A pair of bevel gears is mounted on two intersecting shafts whose shaft angles are at 72° to each other. The velocity ratio of the gears is 2. Find the pitch angles.*

Solution As the velocity ratio is more than 1, the gear is the driver,

$$\tan\gamma_g = \frac{\sin\theta}{\dfrac{\omega_g}{\omega_p} + \cos\theta} = \frac{\sin 72°}{\dfrac{1}{2} + \cos 72°} = 1.176$$

$$\gamma_g = 49.61° \quad \text{or} \quad 49°37'$$

$$\tan \gamma_p = \frac{\sin \theta}{\frac{\omega_p}{\omega_g} + \cos \theta} = \frac{\sin 72°}{2 + \cos 72°} = 0.412$$

$\gamma_p = 22.39°$ or $\underline{22°23'}$

As a check, $\gamma_g + \gamma_p = 49°37' + 22°23' = 72°$

Summary

1. Gears are used to transmit motion from one shaft to another. They use no intermediate link or connector and transmit the motion by direct contact.
2. The speeds of two discs rolling together without slipping are inversely proportional to the radii of the discs.
3. Rotary motion between two parallel shafts is equivalent to the rolling of two cylinders, the motion between two intersecting shafts is equivalent to the rolling of two cones and the motion between two skew shafts is equivalent to the rolling of two hyperboloids with sliding.
4. Spur gears have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load.
5. Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is a plane.
6. In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle, but have teeth of opposite hands.
7. Axial thrust which occurs in case of single-helical gears is eliminated in double-helical gears.
8. When teeth formed on the cones are straight, the gears are known as *straight bevel* and when inclined, they are known as *spiral* or *helical bevel*.
9. Worm gear is a special case of a spiral gear in which the larger wheel, usually, has a hollow or concave shape such that a portion of the pitch diameter of the other gear is enveloped on it.
10. *Pitch circle* is the circle corresponding to a section of the equivalent pitch cylinder by a plane normal to the wheel axis.
11. *Pitch point* is the point of contact of two pitch circles.
12. *Circular pitch* is the distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth.
13. *Module* is the ratio of the pitch diameter in mm to the number of teeth.
14. *Pressure angle* or *angle of obliquity* is the angle between the pressure line and the common tangent to the pitch circles.
15. Locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the *path of contact* or the *contact length*.
16. Locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as *arc of contact*.
17. The law of gearing states that for constant angular velocity ratio of the two gears, the common normal at the point of contact of the two mating teeth must pass through the pitch point.
18. Common forms of teeth that satisfy the law of gearing are *cycloidal* profile teeth and *involute* profile teeth.
19. Owing to the ease of standardization and manufacture, and low cost of production, the use of *involute* teeth has become universal by entirely superseding the *cycloidal* shape.
20. The gears are interchangeable if they are standard ones.
21. Path of contact = path of approach + path of recess

    $= [\sqrt{R_a^2 - R^2 \cos^2 \varphi} - R \sin\varphi]$

    $+ [\sqrt{r_a^2 - r^2 \cos^2 \varphi} - r \sin\varphi]$
22. Art of contact $= \dfrac{\text{Path of contact}}{\cos\varphi}$
23. Mating of two non-conjugate (non-involute) teeth is known as *interference* because the two teeth do not mesh properly and rough action and binding occurs.
24. The minimum number of teeth on the wheel for the given values of the gear ratio, the pressure angle and the addendum coefficient a_w is given by

    $$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \varphi} - 1}$$

25. A gear having its material removed to avoid interference is said to be *undercut* and the process, *undercutting*.
26. *Helix angle* is the angle at which the teeth are inclined to the axis of a gear.
27. *Axial pitch* is the distance between corresponding points on adjacent teeth measured along the direction of the axis.
28. *Lead* is the distance by which a helix advances along the axis of the gear for one turn around.
29. Maximum efficiency of worm gear

$$\eta_{max} = \frac{1-\sin\varphi}{1+\sin\varphi}$$

Exercises

1. What type of gears are used for parallel, intersecting and skew shafts? Explain.
2. Give a detailed classification of gears.
3. What is the difference between double-helical and herringbone gears?
4. What is a worm and worm wheel? Where is it used?
5. Define the terms:
   (i) Pitch circle
   (ii) Pitch diameter
   (iii) Pitch point
   (iv) Circular pitch
   (v) Module
6. Explain the terms addendum and dedendum. What is clearance?
7. Sketch two teeth of a gear and show the following: face, flank, top land, bottom land, addendum, dedendum, tooth thickness, space width, face width and circular pitch.
8. What is pressure line and pressure angle of a gear?
9. State and derive the law of gearing.
10. Deduce an expression for velocity of sliding in a gear drive.
11. What are the main tooth profiles of gear teeth which fulfill the law of gearing? Compare them.
12. Make a comparison of cycloidal and involute tooth forms.
13. What is standard system of gears? How doe it ensure interchangeability of gears?
14. What is path of contact? Derive the relation for its magnitude.
15. Define arc of contact and deduce the expression to find its magnitude.
16. How do you find the number of teeth in contact of two mating gears?
17. What is meant by interference in involute gears? Explain.
18. Derive a relation for minimum number of teeth on the gear wheel and the pinion to avoid interference.
19. Find the minimum number of teeth on a pinion of standard addendum and 20° pressure angle to avoid interference between a rack and pinion.
20. What do you mean by undercutting of gears?
21. How do you differentiate between left-handed and right-handed helical or spiral gears?
22. Define the terms related to helical gears: helix angle, circular pitch, normal circular pitch.
23. Find relations to determine velocity ratio and centre distance of helical gears.
24. Deduce expression for the maximum efficiency of spiral gears.
25. Define the terms related to worm and worm gears: axial pitch, lead and lead angle.
26. How are the centre distance and efficiency of worm gears found?
27. Find relations to calculate the pitch angles of bevel gears.
28. A reduction gear supported on bearings on either side transmits 90 kW of power. The pinion has a pitch circle diameter of 180 mm and rotates at 600 rpm. Determine the maximum force applied due to power transmitted if the pressure angle of the involute teeth is 20°.

    (16.937 kN)
29. The velocity ratio of two spur gears in mesh is 0.4 and the centre distance is 75 mm. For a module of 1.2 mm, find the number of teeth of the gears. What will be the pitch line velocity if the pinion speed is 800 rpm? Also, find the speed of the gear wheel.

    (36,90; 1810 mm/s; 320 rpm)
30. A spur gear has 30 teeth and a module of 1.4 mm. It rotates at 360 rpm. Determine its circular pitch and pitch line velocity.

    (4.4 mm; 791.7 mm/s)
31. Two meshing spur gears with 20° pressure angle have a module of 4 mm. The centre distance is 220 mm and the number of teeth on the pinion is 40. To what value should the centre distance be increased

so that the pressure angle is increased to 22°?

(222 mm)

32. A pinion has 24 teeth and drives a gear with 64 teeth. The teeth are of involute type with 20° pressure angle. The addendum and the module are 8 mm and 10 mm respectively. Determine path of contact, arc of contact and the contact ratio

(41.08 mm, 43.72, 1.39)

33. Two gears in mesh have a module of 10 mm and a pressure angle of 25°. The pinion has 20 teeth and the gear has 52. The addendum on both the gears is equal to one module. Determine the
    - (i) number of pairs of teeth in contact
    - (ii) angles of action of the pinion and the wheel
    - (iii) ratio of the sliding velocity to the rolling velocity at the pitch point and at the beginning and end of engagement

(1.475; $\delta_p = 26°36'$, $\delta_g = 10°13'$; zero, 0.304, 0.278)

34. The number of teeth on the gear and the pinion of two spur gears in mesh are 30 and 18 respectively. Both the gears have a module of 6 mm and a pressure angle of 20°. If the pinion rotates at 400 rpm, what will be the sliding velocity at the moment the tip of the tooth of pinion has contact with the gear flank? Take addendum equal to one module. Also, find the maximum velocity of sliding.

(908 mm/s; 981.5 mm/s)

35. Two 20° involute spur gears have a module of 6 mm. The larger wheel has 36 teeth and the pinion has 16 teeth. If the addendum be equal to one module, will the interference occur? What will be the effect if the number of teeth on the pinion is reduced to 14?

(No; interference occurs)

36. The addendum on each wheel of two mating gears is to be such that the line of contact on each side of the pitch point is half the maximum possible length. The number of teeth on the two gears is 24 and 48. The teeth are of 20° pressure angle involute with a module of 12 mm. Determine the addendum for the pinion and the gear. Also, find the arc of contact and the contact ratio.

(23.4 mm, 9.3 mm, 78.6 mm, 2.08)

37. The following data refer to two meshing gears having 20° involute teeth:

    Number of teeth of gear wheel = 52

    Number o teeth of pinion =20

    Speed of pinion = 360 rpm

    Module = 8 mm

    If the addendum of each gear is such that the path of approach and path of recess are half of their maximum possible values, determine the addendum for the gear and the pinion and the length of arc of contact.

(5.07 mm; 18.04 mm; 52.4 mm)

38. Determine the minimum number of teeth and the arc of contact (in terms of module) to avoid interference in the following cases:

    (a) Gear ratio is unity.

    (b) Gear ratio is 3.

    (c) Pinion gears with a rack.

    The addendum of the teeth is 0.88 module and the power component is 0.94 times the normal thrust.

(11, 3.96 m; 14, 4.48 m; 16, 4.24 m)

39. Two 20° involute spur gears having a velocity ratio of 2.5 mesh externally. The module is 4 mm and the addendum is equal to 1.23 module. The pinion rotates at 150 rpm. Find the
    - (i) minimum number of teeth on each wheel to avoid interference
    - (ii) number of pairs of teeth in contact.

(45, 18; 1.95)

40. If the angle of obliquity of a pair of gear wheels is 20°, and the arc of approach or recess not less than the pitch, what will be the least number of teeth on the pinion?

(18)

41. Two 20° full-depth involute spur gears having 30 and 48 teeth are in mesh. The pinion rotates at 800 rpm. The module is 4 mm. Find the sliding velocities at the engagement and at the disengagement of a pair of teeth and the contact ratio. If the interference is just avoided, find (i) the addenda on the wheel and the pinion, (ii) the path of contact, (iii) the maximum velocity of sliding at engagement and disengagement of a pair of teeth, and (iv) contact ratio.

(8.8 mm, 17.6 mm; 53.35 mm; 2.934 m/s, 4.695 m/s; 4.52)

42. A rack is driven by a pinion having 24 involute teeth and a 140 mm pitch circle diameter. The addendum of both pinion and the rack is 6 mm. Determine the least pressure angle which can be used to avoid interference. For this pressure angle, find the minimum number of teeth in contact at a time.

(17.02°, 2.11)

43. The centre distance between two meshing spiral gears is 150 mm and the angle between the shafts is 60°. The gear ratio is 2 and the normal circular pitch is 10 mm. The driven gear has a helix angle of 25° determine the

(i) number of teeth on each wheel
(ii) exact centre distance
(iii) efficiency if the friction angle is 4°

(28, 56; 152.7 mm; 0.92)

44. Two right-handed helical gears connect two shafts 70° apart. The larger gear has 50 teeth and the smaller has 20. If the centre distance is 167 mm, determine the helix angle of the gears. The normal module is 4 mm.

(28°; 42°)

45. The angle between two shafts is 90°. They are joined by two spiral gears having a normal circular pitch of 6 mm and a gear ratio of 2. if the approximate distance between the shafts is 200 mm and the friction angle is 6°, determine the following for the maximum efficiency of the drive:
(a) Number of teeth
(b) Centre distance (exact)
(c) Pitch diameters
(d) Efficiency

(50, 100; 199.85 mm; 142.7 mm, 257 mm; 0.81)

46. A three-start worm has a pitch diameter of 80 mm and a pitch of 20 mm. It rotates at 600 rpm and drives a 40-tooth worm gear. If coefficient of friction is 0.05, find the
(i) helix angle of the worm
(ii) speed of the gear
(iii) centre distance
(iv) efficiency and maximum efficiency

(76°56′; 45 rpm; 167.3 mm; 0.817; 0.905)

47. Two meshing bevel gears are mounted on two intersecting shafts, the angle between the shafts being 48°. The velocity ratio of the gears is 2.4. Determine the pitch angles.

(34.39°, 13.61°)

11 GEAR TRAINS

Introduction

A gear train is a combination of gears used to transmit motion from one shaft to another. It becomes necessary when it is required to obtain large speed reduction within a small space. The following are the main types of gear trains:

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Planetary or epicyclic gear train

In *simple* gear trains, each shaft supports one gear. In a *compound* gear train, each shaft supports two gear wheels except the first and the last. In a *reverted* gear train, the driving and the driven gears are coaxial or coincident. In all these three types, the axes of rotation of the wheels are fixed in position and the gears rotate about their respective axes. However, it is also possible that in a gear train, the axes of some of the wheels are not fixed but rotate around the axes of other wheels with which they mesh. Such trains are known as *planetary* or *epicyclic* gear trains. Epicyclic gear trains are useful to transmit very high velocity ratios with gears of smaller sizes in a lesser space.

11.1 SIMPLE GEAR TRAIN

A series of gears, capable of receiving and transmitting motion from one gear to another is called a simple gear train. In it, all the gear axes remain fixed relative to the frame and each gear is on a separate shaft (Fig. 11.1).

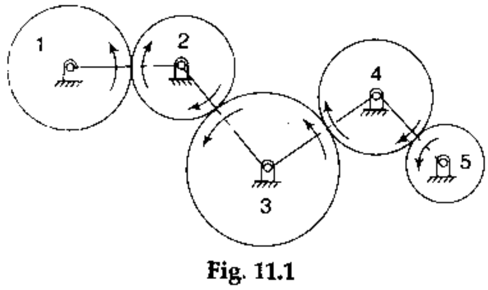


Fig. 11.1

In a simple gear train we can observe the following:

1. Two external gears of a pair always move in opposite directions.
2. All odd-numbered gears move in one direction and all even-numbered gears in the opposite direction. For example, gears 1, 3, 5, etc, move in the counter-clockwise direction.
3. *Speed ratio*, the ratio of the speed of the driving to that of the driven shaft, is negative when the input and the output gears rotate in the opposite directions and it is positive when the two rotate in the same direction. The reverse of the speed ratio is known as the *train value* of the gear train.
4. All the gears can be in a straight line or arranged in a zig-zag manner. A simple gear train can also have bevel gears.

Let T = number of teeth on a gear

N = speed of a gear in rpm.

Refer Fig. 11.1.

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \left[\text{Also } \frac{\omega_2}{\omega_1} = \frac{2\pi N_2}{2\pi N_1} = \frac{N_2}{N_1}\right]$$

and

$$\frac{N_3}{N_2} = \frac{T_2}{T_3}, \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_5}{N_4} = \frac{T_4}{T_5}$$

Multiplying,

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} \times \frac{N_5}{N_4} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} \times \frac{T_4}{T_5}$$

or

$$\text{Train value } \frac{N_5}{N_1} = \frac{T_1}{T_5} = \frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven gear}}$$

$$\text{Speed ratio} = \frac{1}{\text{train value}}$$

or

$$\frac{N_1}{N_5} = \frac{T_5}{T_1} \tag{11.1}$$

Thus, it is seen that the intermediate gears have no effect on the speed ratio and, therefore, they are known as *idlers*.

11.2 COMPOUND GEAR TRAIN

When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity, it is known as compound gear train. In this type, some of the intermediate shafts, i.e., other than the input and the output shafts, carry more than one gear as shown in Fig. 11.2.

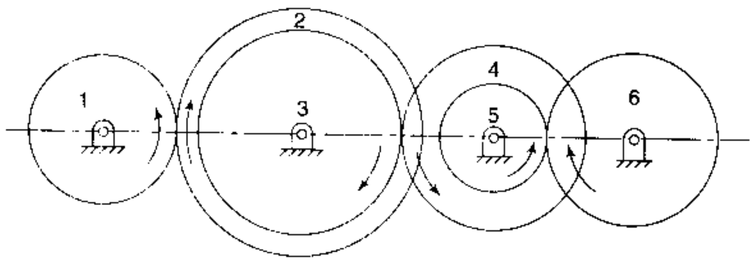
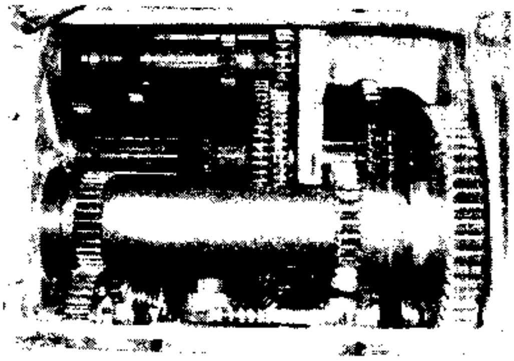


Fig. 11.2

If the gear 1 is the driver then



Gear box of a lathe consisting of compound gears

A compound gear train

$$\frac{N_2}{N_1} = \frac{T_1}{T_2},\ \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_6}{N_5} = \frac{T_5}{T_6}$$

or

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

or

$$\frac{N_2}{N_1} \times \frac{N_4}{N_2} \times \frac{N_6}{N_4} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_6}{N_1} = \frac{T_1}{T_2} \cdot \frac{T_3}{T_4} \cdot \frac{T_5}{T_6} \qquad (11.2)$$

or

$$\text{Train value} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

Example 11.1 *A compound gear train shown in Fig. 11.3 consists of compound gears B-C and D-E. All gears are mounted on parallel shafts. The motor shaft rotating at 800 rpm is connected to the gear A and the output shaft to the gear F. The number of teeth on gears A, B, C, D, E and F are 24, 56, 30, 80, 32 and 72 respectively. Determine the speed of the gear F.*

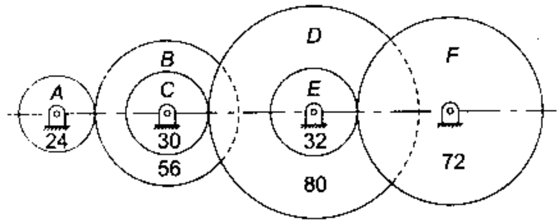


Fig. 11.3

Solution

$$\frac{N_F}{N_A} = \frac{T_A}{T_B}\frac{T_C}{T_D}\frac{T_E}{T_F} = \frac{24}{56} \times \frac{30}{80} \times \frac{32}{72}$$

$= 0.07143$ or $N_F = 0.07143 \times 800 = 57.14$ rpm

11.3 REVERTED GEAR TRAIN

If the axes of the first and the last wheels of a compound gear coincide, it is called a reverted gear train. Such an arrangement is used in clocks and in simple lathes where *back gear* is used to give a slow speed to the chuck.

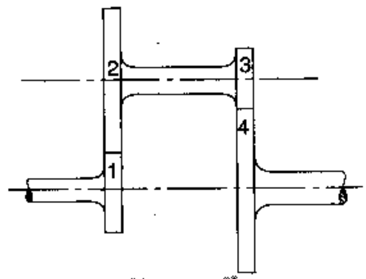


Fig. 11.4

Referring Fig. 11.4,

$$\frac{N_4}{N_1} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

$$= \frac{T_1}{T_2}\frac{T_3}{T_4} \qquad (11.3)$$

Also, if r is the pitch circle radius of a gear,

$$r_1 + r_2 = r_3 + r_4 \qquad (11.4)$$

Example 11.2 *A reverted gear train shown in Fig. 11.4 is used to provide a speed ratio of 10. The module of gears 1 and 2 is 3.2 mm and of gears 3 and 4 is 2 mm. Determine suitable numbers of teeth for each gear. No gear is to have less than 20 teeth. The centre distance between shafts is 160 mm.*

Solution Let us assume that the speed ratio of the pair of gears 1 and 2 = 2.5 or $\frac{N_1}{N_2} = \frac{T_2}{T_1} = 2.5$

and speed ratio of the pair of gears 3 and 4 = 4 or

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} = 4$$

Now, $r_1 + r_2 = r_3 + r_4 = 160$

$$\text{or} \quad \frac{m_1T_1}{2} + \frac{m_2T_2}{2} = 160$$

$$\text{and} \quad \frac{m_3T_3}{2} + \frac{m_4T_4}{2} = 160$$

or $3.2(T_1 + T_2) = 320$ and $2(T_3 + T_4) = 320$

or $T_1 + T_2 = 100$ and $T_3 + T_4 = 160$

or $T_1 + 2.5T_1 = 100$ and $T_3 + 4T_3 = 160$

or $T_1 = 28.57$ say 28 and $T_3 = 32$

To ensure the same centre distance between two sets of gears,

$T_2 = 100 - 28 = 72$ and $T_4 = 160 - 32 = 128$

Exact velocity ratio

$$= \frac{T_1}{T_2} \cdot \frac{T_3}{T_4} = \frac{28 \times 32}{72 \times 128} = 10.29$$

If number of teeth on the gear 1 are taken as 29 then $T_2 = 100 - 29 = 71$

Exact velocity ratio

$$= \frac{T_1}{T_2} \cdot \frac{T_3}{T_4} = \frac{29 \times 32}{71 \times 128} = 9.79$$

11.4 PLANETARY OR EPICYCLIC GEAR TRAIN

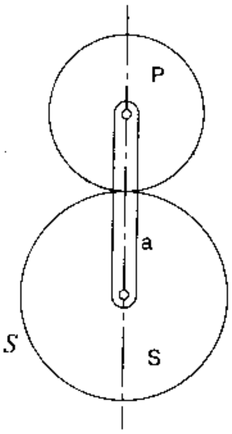


Fig. 11.5

A gear train having a relative motion of axes is called a *planetary* or an *epicyclic gear train* (or simply epicyclic gear or train). In an epicyclic train, the axis of at least one of the gears also moves relative to the frame.

Consider two gear wheels *S* and *P*, the axes of which are connected by an arm *a* (Fig. 11.5). If the arm *a* is fixed, the wheels *S* and *P* constitute a simple train. However, if the wheel *S* is fixed so that the arm can rotate about the axis of *S*, the wheel *P* would also move around *S*. Therefore, it is an epicyclic train.

Usually, the wheel *P* is known as the *epicyclic wheel*. The term epicyclic emerges from the fact that the wheel *P* rolls outside another wheel and traces an epicyclic path. It is also possible that the fixed wheel is annular and another wheel rolls inside it. In that case, the path traced will be a hypocycloid. However, it has become customary to call all gears, in which one of the axes rotates about a fixed axis, as epicyclic gears.

Large speed reductions are possible with epicyclic gears and if the fixed wheel is annular, a more compact unit could be obtained. Important applications of epicyclic gears are in transmission, computing devices, and so on.

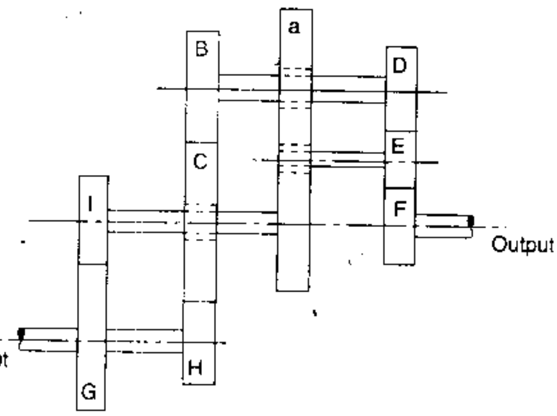


Fig. 11.6

In an epicyclic gear, one wheel is, usually, fixed as in the above case. However, it is not necessary at all and the wheel S can have rotations in any direction about its axis, i.e., clockwise or counter-clockwise. Figure 11.6 shows an epicyclic gear with no fixed wheel. The epicyclic gear consists of wheels B, C, D, E and F and the arm a. Wheels G and H are merely the drivers; G drives the arm a through the wheel I whereas H drives the wheel C.

In general, gear trains have two degrees of freedom. This means to obtain a controlled motion of the output, the train must have two inputs. In Fig. 11.6, two inputs, to the wheel C and the arm a result in a definite motion of the wheel F or of the output shaft. However, number of inputs can be reduced to one, if one wheel of the train is fixed. That amounts to reducing the speed of that gear wheel to zero.

In dealing with epicyclic gears, distinction must be made between wheels which are part of the epicyclic train and those which are not. In the gear train of Fig. 11.6, G and H cannot be the parts of the epicyclic train. Also, the speed of the arm a will be the same as that of the wheel I.

11.5 ANALYSIS OF EPICYCLIC GEAR TRAIN

Epicyclic trains usually have complex motions. Therefore, comparatively simple methods are used to analyse them which do not require accurate visualization of the motions.

Refer Fig. 11.5 and assume that the arm a is fixed. Turn S through x revolutions in the clockwise direction. Assuming clockwise motion of a wheel as positive and counter-clockwise as negative.

revolutions made by $a = 0$ (Arm a fixed)

revolutions made by $S = x$

revolutions made by $P = -(T_s/T_p)x$

Now, if the mechanism is locked together and turned through a number of revolutions, the relative motions between a, S and P will not alter. Let the locked system be turned through y revolutions in the clockwise direction. Then

revolutions made by $a = y$

revolutions made by $S = y + x$

revolutions made by $P = y - \left(\frac{T_s}{T_p}\right)x$

This implies that if the arm a turns through y revolutions and S through $(y - x)$ revolutions in the same direction then P will rotate through $[y - (T_s/T_p)x]$ revolutions in space or relative to the fixed axis of S. Thus, if revolutions made by any of the two elements are known, x and y can be solved and the revolutions made by the third can be determined. Thus, the procedure can be summarized as follows:

1. Lock the arm and assume the other wheels free to rotate.
2. Turn any convenient gear through one revolution in the clockwise direction and record the number of revolutions made by each of the other wheels.
3. Multiply all the above recordings by x and write the same in the second row. This is equivalent to the statement that the chosen wheel is given x revolutions in the clockwise direction keeping the arm fixed.
4. Add y to all the quantities in the second row and make the recordings in the third row. This amounts to the fact that by locking the whole system, it is turned through y revolutions in the clockwise direction. Thus, the arm makes y revolutions, the chosen wheel $(y + x)$ revolutions, and so on.
5. Apply the given conditions and find the values of x and y. Having known x and y, the revolutions made by any of the wheels can be known.

The above procedure is also illustrated by the Table 11.1 for Fig. 11.5.

Table 11.1

Line	Action	Revs. of a	Revs. of S	Revs. of P
1.	a fixed, S + 1 rev.	0	1	$-\frac{T_S}{T_P}$
2.	a fixed, S + x rev.	0	x	$-\frac{T_S}{T_P}x$
3.	Add y	y	y + x	$y-\frac{T_S}{T_P}x$

Note that the number of revolutions of the wheel P given in the third row of the table is the number of revolutions in space or relative to the fixed axis of S and not about its own axis.

Consider a system shown in Fig. 11.7 in which a is an arm which can rotate about a fixed axis O and P is a wheel which has its axis at the other end of a. For the moment, assume the wheel P to be fixed to the arm a. Now turn the arm through 1/4 revolution, i.e., from the position 1 to the position 2. It will be observed that the wheel P has also turned through 1/4 revolution. But the rotation of P is not about its own axis, as it is fixed to the arm. Similarly, it can be seen that P also rotates through one revolution as the arm turns through one revolution. However, the rotation of P is in space or about the axis of rotation of the arm and not about its own axis. Thus, if the arm makes y revolutions about O, the wheel P also rotates through y revolutions. This suggests that the number of revolutions of P about its own axis can be obtained by subtracting the number of revolutions of the arm from the total number of revolutions of P, or

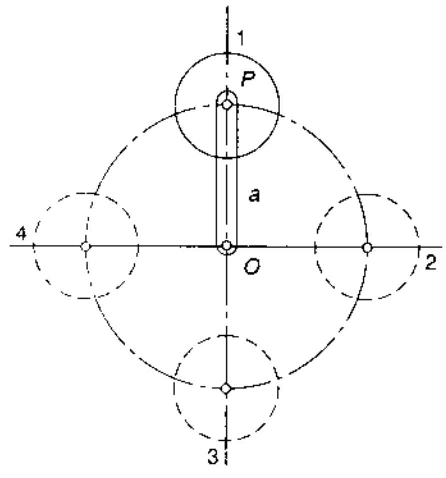


Fig. 11.7

Rev. of P about its own axis = total revs. about axis of arm − revs. of the arm

$$= \left(y-\frac{T_S}{T_P}x\right)-y$$

$$= -\frac{T_S}{T_P}x \qquad (11.5)$$

Relative Velocity Method

Angular vel. of S = angular vel. of S rel. to a + angular vel. of a

or $\omega_s = \omega_{sa} + \omega_a$

or $N_s = N_{sa} + N_a$

Similarly, $N_P = -N_{Pa} + N_a$ (N_s and N_p are to be in opposite directions)

$\therefore$ $N_{sa} = N_s - N_a$

and $N_{Pa} = N_a - N_p$

or $$\frac{N_{sa}}{N_{Pa}} = \frac{N_s - N_a}{N_a - N_p}$$

or $$\frac{T_P}{T_s} = -\frac{N_s - N_a}{N_P - N_a} \quad (11.6)$$

Usually, T_P/T_s and out of N_s, N_a and N_P, two are known and the third is calculated.

Example 11.3 *An epicyclic gear train consists of an arm and two gears A and B having 30 and 40 teeth respectively. The arm rotates about the centre of the gear A at a speed of 80 rpm counter-clockwise. Determine the speed of the gear B if (i) the gear A is fixed, and (ii) the gear A revolves at 240 rpm clockwise instead of being fixed.*

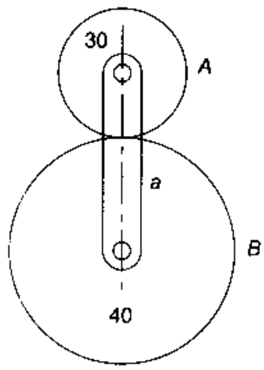


Fig. 11.8

Solution Refer Fig. 11.8. Prepare Table 11.2.

Considering counter-clockwise as positive direction,

(i) Gear A is fixed, thus $y + x = 0$

Arm a rotates at 80 rpm, $y = 80$

$\therefore$ $x = -80$

Speed of the gear B, $y - \frac{3}{4}x$

$80 - \frac{3}{4} \times (-80) = 140 =$ rpm (counter-clockwise)

(i) Gear A revolves at 240 rpm clockwise,

$y + x = -240$

$\therefore$ $x = -80 - 240 = -320$

Speed of the gear B, $y - \frac{3}{4}x$

$= 80 - \frac{3}{4} \times (-320)$

$= 320$ rpm (counter-clockwise)

Algebraic Method

(i) $$\frac{T_A}{T_B} = -\frac{N_B - N_a}{N_A - N_a}$$

or $$-\frac{30}{40} = \frac{N_B - 80}{0 - 80} \text{ or } N_B = 140 \text{ rpm}$$

(ii) $$-\frac{30}{40} = \frac{N_B - 80}{-240 - 80} \text{ or } N_B = 320 \text{ rpm}$$

Table 11.2

Line	*Action*	*Revs. of a*	*Revs. of A*	*Revs. of B*
1.	a fixed, S + 1 rev.	0	1	$-\frac{30}{40}$
2.	a fixed, S + x rev.	0	x	$-\frac{3}{4}x$
3.	Add y	y	$y + x$	$y - \frac{3}{4}x$

Example 11.4 *Figure 11.9 shows a gear train in which gears B and C constitute a compound gear. The number of teeth are shown along with each wheel in the figure. Determine the speed and the direction of rotation of wheels A and E if the arm revolves at 210 rpm clockwise and the gear D is fixed.*

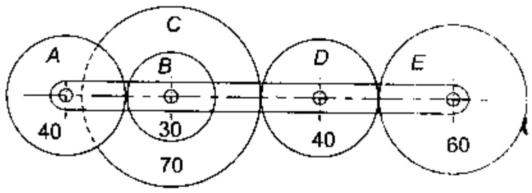


Fig. 11.9

Solution Prepare the Table 11.3:

For given conditions,

Arm a rotates at 210 rpm clockwise, $y = 210$

$$\text{Gear } D \text{ is fixed, thus } y + \frac{7x}{3} = 0$$

or $$210 + \frac{7x}{3} = 0 \text{ or } x = -90$$

Speed of $A = y + x = 210 - 90 = 120$ rpm (clockwise)

$$\text{Speed of } E = y - \frac{14x}{9} = 210 - \frac{14 \times (-90)}{9}$$

$= 350$ rpm (clockwise)

Table 11.3:

Action	a	A	B/C	D	E
'a' fixed, A + 1 rev.	0	1	$-\frac{40}{30}$	$-\frac{40}{30} \times \left(-\frac{70}{40}\right)$	$-\frac{7}{3} \times \frac{40}{60}$
'a' fixed, A + x rev.	0	x	$-\frac{40x}{30}$	$\frac{7x}{3}$	$-\frac{14x}{9}$
Add y	y	$y + x$	$y - \frac{40}{30}$	$y + \frac{7x}{3}$	$y - \frac{14x}{9}$

Example 11.5 *An epicyclic gear train is shown in Fig. 11.10. The number of teeth on A and B are 80 and 200. Determine the speed of the arm a*

(i) if A rotates at 100 rpm clockwise and B at 50 rpm counter-clockwise

(ii) if A rotates at 100 rpm clockwise and B is stationary

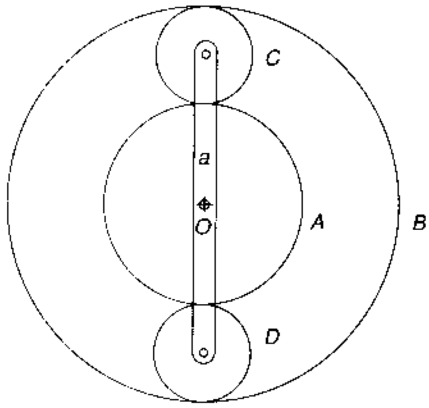


Fig. 11.10

Solution $T_A = 80$ $T_B = 200$

Now, $$T_B = 2\left[\frac{T_A}{2} + T_C\right]$$

or $$200 = 2\left[\frac{80}{2} + T_C\right]$$

or $$T_C = 60$$

Prepare Table 11.4:

Table 11.4

Action	*a*	*A*	*C/D*	*B*
a fixed, A + 1 rev.	0	1	$-\frac{80}{60}$	$-\frac{80}{60}\times\frac{60}{200}$
a fixed, A + x rev.	0	x	$-\frac{4x}{3}$	$-\frac{2x}{5}$
Add y	y	$y+x$	$y-\frac{4x}{3}$	$y-\frac{2x}{5}$

(i) From the given conditions,

$$y + x = 100 \quad \text{or} \quad y = 100 - x$$

and $$y - \frac{2x}{5} = 50$$

or $$100 - x - \frac{2x}{5} = -50$$

or $$x = 107.1 \quad \text{and } y = -7.1$$

Thus, speed of arm a = 7.1 rpm counter-clockwise.

(ii) $$y + x = 100 \qquad \text{or } y = 100 - x$$

and $$y - \frac{2x}{5} = 0$$

or $$100 - x - \frac{2x}{5} = 0$$

or $$x = 71.4 \quad \text{and } y = 28.6$$

Thus, speed of arm a = 28.6 rpm clockwise.

Example 11.6 *In the epicyclic gear train shown in Fig. 11.11, the compound wheels A and B as well as internal wheels C and D rotate independently about the axis O. The wheels E and F rotate on the pins fixed to the arm a. All the wheels are of the same module. The number of teeth on the wheels are*

$$T_A = 52, T_B = 56, T_E = T_F = 36$$

Determine the speed of C if

(i) the wheel D fixed and arm a rotates at 200 rpm clockwise

(ii) the wheel D rotates at 200 rpm counter-clockwise and the arm a rotates at 20 rpm counter-clockwise

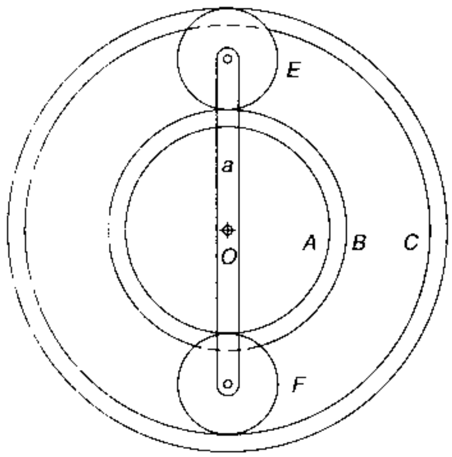


Fig. 11.11

Solution $T_A = 52 \qquad T_B = 56 \qquad N_F = N_P = 36$

Now, $$T_D = 2\left[\frac{T_B}{2} + T_E\right] = 2\left[\frac{56}{2} + 36\right] = 128$$

$$T_C = 2\left[\frac{T_A}{2} + T_F\right] = 2\left[\frac{52}{2} + 36\right] = 124$$

Prepare Table 11.5.

Table 11.5

Action	a	A/B	E	F	C	D
'a' fixed, A + 1 rev.	0	1	$-\frac{56}{36}$	$-\frac{52}{36}$	$\frac{52}{36}\times\frac{36}{124}$	$\frac{56}{36} X \frac{36}{128}$
'a' fixed, A + x rev.	0	x	$-\frac{14x}{9}$	$-\frac{13x}{9}$	$\frac{13x}{31}$	$\frac{7x}{16}$
Add y	y	y + x	$y-\frac{14x}{9}$	$y-\frac{13x}{9}$	$y-\frac{13x}{31}$	$y-\frac{7x}{16}$

(i) From given conditions,

Arm a rotates at 200 rpm clockwise,

$\therefore y = 200$

$$D \text{ is fixed, } \therefore\ y-\frac{7x}{16}=0$$

or $$200-\frac{7x}{16}=0$$

or $$x = 457.1$$

$$\text{Speed of } C = y-\frac{13x}{31}$$

$$= 200-\frac{13\times 457.1}{31}$$

$$= \underline{8.31 \text{ rpm}} \text{ (clockwise)}$$

(ii) $y = 200$,

$$\therefore\ y-\frac{7x}{16} = -20$$

or $$200-\frac{7x}{16} = -20$$

or $$x = 502.86$$

$$\text{Speed of } C = y-\frac{13x}{31}$$

$$= 200-\frac{13\times 502.86}{31}$$

$$= -10.9 \text{ rpm}$$

or $\underline{10.9 \text{ rpm}}$ counter-clockwise

Example 11.7 *The annulus A in the gear shown in Fig. 11.12 rotates at 300 rpm about the axis of the fixed wheel S which has 80 teeth. The three-armed spider (only one arm a is shown in Fig.11.12a) is driven at 180 rpm. Determine the number of teeth required on the wheel P.*

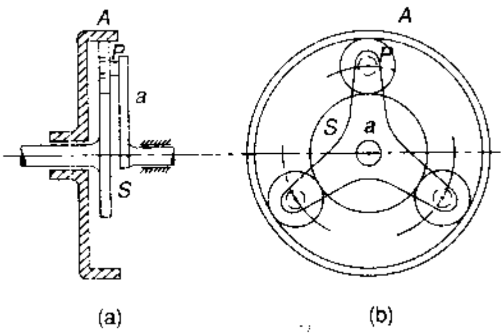


Fig. 11.12

Solution: $N_s = 0$ $N_A = 300$ rpm

$N_a = 180$ rpm $T_S = 80$

Prepare Table 11.6.

Table 11.6

Action	*a*	*S*	*P*	*A*
'a' fixed, S + 1 rev.	0	1	$-\dfrac{80}{T_P}$	$-\dfrac{80}{T_P}\times\dfrac{T_P}{T_A}=-\dfrac{80x}{T_A}$
'a' fixed, S + x rev.	0	x	$-\dfrac{80x}{T_P}$	$-\dfrac{80x}{T_A}$
All given y revs.	y	$y+x$	$y-\dfrac{80x}{T_P}$	$y-\dfrac{80x}{T_A}$

From given conditions,

$$N_a = y = 180 \quad \text{(i)}$$

$$N_S = y + x = 0 \quad \text{(ii)}$$

$$N_A = y - \frac{80x}{T_A} = 300 \quad \text{(iii)}$$

From (i) and (ii), $x = -y = -180$

Solving (iii),

$$180 - \frac{80(-180)}{T_A} = 300$$

or $$\frac{14\,400}{T_A} = 120$$

or $T_A = 120$

The pitch diameters of the wheels are proportional to the number of teeth on them.

$$T_S + 2T_P = T_A$$

or $80 + 2T_P = 120$ or $T_P = 20$

***Example* 11.8** *In a reduction gear shown in Fig.11.13, the input S has 24 teeth. P and C constitute a compound planet having 30 and 18 teeth respectively. If all the gears are of the same pitch, find the ratio of the reduction gear. Assume A to be fixed.*

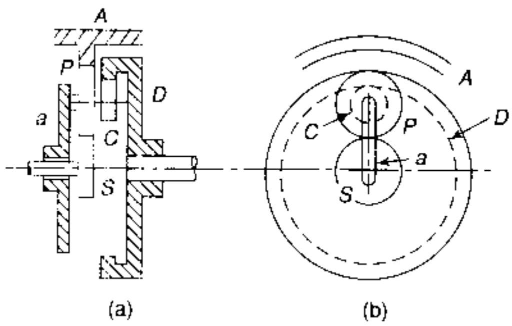


Fig. 11.13

Solution

$T_S = 24$

$T_P = 30$

$T_C = 18$

$N_A = 0$

Pitch diameters of wheels are proportional to the number of teeth on them.

$$T_A = 2\left[\frac{T_S}{2} + T_P\right] = 2\left[\frac{24}{2} + 30\right] = 84$$

$$T_D = 2\left[\frac{T_S}{2} + \frac{T_P}{2} + \frac{T_C}{2}\right] = 2\left[\frac{24}{2} + \frac{30}{2} + \frac{18}{2}\right] = 72$$

Prepare Table 11.7.

Table 11.7

Action	*a*	*S*	*P/C*	*A*	*D*
'*a*' fixed, $S + 1$ rev.	0	1	$-\frac{24}{30}$	$-\frac{24}{30}\times\frac{30}{84}$	$-\frac{24}{30}\times\frac{18}{72}$
'*a*' fixed, $S + x$ rev.	0	x	$-\frac{4x}{5}$	$\frac{-2x}{7}$	$-\frac{x}{5}$
Add y	y	$y+x$	$y-\frac{4x}{5}$	$y-\frac{2x}{7}$	$y-\frac{x}{5}$

From given conditions,

$$N_A = y - \frac{2x}{7} = 0 \text{ or } y = \frac{2x}{7}$$

$$\therefore \quad \frac{N_S}{N_D} = \frac{y+x}{y-\frac{x}{5}} = \frac{\frac{2x}{7}+x}{\frac{2x}{7}-\frac{x}{5}} = \frac{9}{7}\times\frac{35}{3} = 15$$

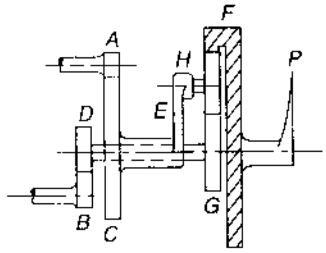


Fig. 11.14

***Example* 11.9** *Figure 11.14 shows a port indicator for a twin-screw ship. It is found that the pointer P remains stationary if the propellers run at the same speed and drive the gears C and D in the same direction through equal gears A and B. If the number of teeth on G and F are 24 and 50 respectively, find the ratio of the number of teeth on C to that on D.*

What will be the speed of the pointer if B runs at 5% faster than A and if the speed of C is 100 rpm?

Solution: $T_G = 24 \quad T_F = 50 \quad N_F = N_P = 0$

Now, $$T_F = 2\left[\frac{T_G}{2} + T_H\right]$$

or $$50 = 2\left[\frac{24}{2} + T_H\right]$$

or $$T_H = \frac{50-24}{2} = 13$$

Prepare Table 11.8.

Table 11.8

Action	*C/E*	*D/G*	*H*	*F/P*
C fixed, $D + 1$ rev.	0	1	$-\frac{24}{13}$	$-\frac{24}{13}\times\frac{13}{50}$
C fixed, $D + x$ rev.	0	x	$-\frac{24x}{13}$	$-\frac{12x}{25}$
Add y	y	$y+x$	$y-\frac{24x}{13}$	$y-\frac{12x}{25}$

Given,

$$N_F = y - \frac{12x}{25} = 0 \text{ or } y = \frac{12x}{25}$$

$$\therefore \quad \frac{T_C}{T_D} = \frac{N_D}{N_C} = \frac{y+x}{y} = \frac{\frac{12x}{25}+x}{\frac{12x}{25}} = \frac{37}{12}$$

When B and C rotate at different speeds,

$$N_C = y = 100$$

$$N_A = 100 \times \frac{T_C}{T_A}$$

$$N_B = \left(100 \times \frac{T_C}{T_A}\right) \times 1.05$$

$$N_D = N_B \times \frac{T_B}{T_D} = \left(100 \times \frac{T_C}{T_A} \times 1.05\right) \times \frac{T_B}{T_D}$$

$$= 100 \times 1.05 \times \frac{T_C}{T_D} \qquad (\text{as } T_A = T_B \text{ given})$$

$$= 100 \times 1.05 \times \frac{37}{12}$$

or $\quad y + x = 323.75$

$$x = 323.75 - 100 = 223.75$$

$$\therefore \quad N_P = y - \frac{12x}{25}$$

$$= 100 - \frac{12 \times 223.75}{25} = -7.4 \text{ rpm}$$

i.e., P rotates at 7.4 rpm in direction opposite to that of C.

11.6 TORQUES IN EPICYCLIC TRAINS

Torques are transmitted from one element to another when a geared system transmits power. Assume that all the wheels of a gear train rotate at uniform speeds, i.e., accelerations are not involved. Also, each wheel is in equilibrium under the action of torques acting on it.

Let N_S, N_a, N_P and N_A be the speeds and $\mathbf{T}_S$, $\mathbf{T}_a$, $\mathbf{T}_P$ and $\mathbf{T}_A$ the torques transmitted by S, a, P and A respectively (Fig. 11.15).

We have,

$\mathbf{T} = 0$ (T is the torque)

or $\quad \mathbf{T}_S + \mathbf{T}_a + \mathbf{T}_P + \mathbf{T}_A = 0 \qquad (11.7)$

Now S and a are connected to machinery outside the system and thus transmit external torques. The planet P can rotate on its own pin fixed to a but is not connected to anything outside. Therefore, it does not transmit external torque. The annulus A is either locked by an external torque or transmits power or torque either to or from the system through external teeth.

Therefore, Eq. (11.7) becomes,

$\mathbf{T}_S + \mathbf{T}_a + 0 + \mathbf{T}_A = 0$

or $\quad \mathbf{T}_S + \mathbf{T}_a + \mathbf{T}_A = 0 \qquad (11.8)$

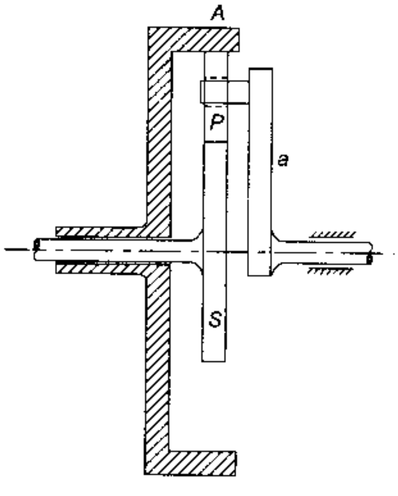


Fig. 11.15

If A is fixed, $\mathbf{T}_A$ is usually known as the *braking* or the *fixing torque*. Out of $\mathbf{T}_S$ and $\mathbf{T}_a$ one will be the driving torque and the other, the output or the resisting torque.

Assuming no losses in power transmission,

$\Sigma \mathbf{T}\omega = 0$

or $\quad \Sigma \mathbf{T}N = 0$

or $\quad \mathbf{T}_S N_S + \mathbf{T}_a N_a + \mathbf{T}_A N_A = 0 \qquad (11.9)$

If A is fixed, $N_A = 0$

and

$$\mathbf{T}_S N_S + \mathbf{T}_a N_a = 0 \qquad (11.10)$$

Proper directions of speeds and torques must be taken into account.

Example 11.10 *Figure 11.16 shows a gear train in which gears D-E and F-G are compound gears. D gears with A and B; E gears with F; and G gears with C. The numbers of teeth on each gear are A = 60, B = 120, C = 135, D = 30, E = 75, F = 30, G = 60. If the wheel A is fixed and the arm makes 20 revolutions clockwise, find the revolutions of B and C.*

If the arm is applied a turning moment of 1 kN.m, determine the turning moment on the shaft supporting the wheel C.

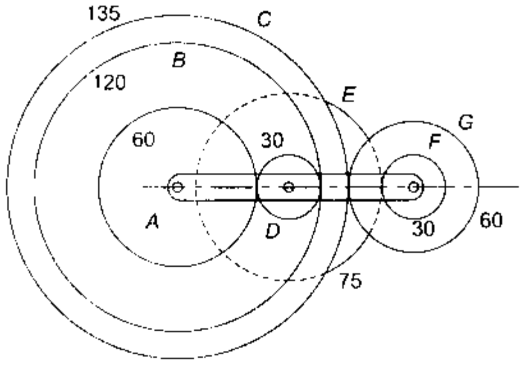


Fig. 11.16

Solution Prepare Table 11.9.

For given conditions,

Arm a rotates at 20 rpm clockwise, $y = 20$

Gear A is fixed, thus $y + x = 0$ or $x = -20$

Speed of $B = y - \frac{x}{2} = 20 - \frac{-20}{2} = 30$ rpm (clockwise)

Speed of $C = y - \frac{20x}{9} = 20 - \frac{20 \times (-20)}{9}$

$= 64.4$ rpm (clockwise)

$\mathbf{T}_c N_c + \mathbf{T}_a N_a = 0$ or $T_c \times 64.4 + 1 \times 20 = 0$

or $T_c = -3.22$ kN.m

Thus, turning moment on shaft of the gear C

$= 3.22$ kN.m

Example 11.11 *The number of teeth in the gear shown in Fig. 11.17 are as follows:*

$T_S = 18$, $T_P = 24$, $T_C = 12$, $T_A = 72$

P and C form a compound gear carried by the arm a and the annular gear A is held stationary. Determine the speed of the output at a. Also find the holding torque required on A if 5kW is delivered to S at 800 rpm with an efficiency of 94%.

In case the annulus A rotates at 100 rpm in the same direction as S, what will be the new speed of a?

Table 11.9

Action	*a*	*A*	*D/E*	*B*	*F/G*	*C*
'a' fixed, A + 1 rev.	0	1	$-\frac{60}{30}$	$-\frac{60}{30}\times\frac{30}{120}$	$-\frac{60}{30}\cdot\left(-\frac{75}{30}\right)=5$	$5\times\left(-\frac{60}{135}\right)$
'a' fixed, $A + x$ rev.	0	x	$-2x$	$-\frac{x}{2}$	$5x$	$-\frac{20x}{9}$
Add y	y	$y + x$	$y - 2x$	$y - \frac{x}{2}$	$y + 5x$	$y - \frac{20x}{9}$

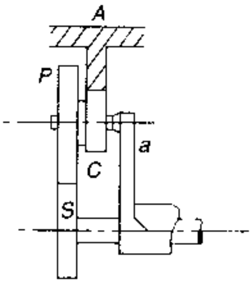


Fig. 11.17

Solution

$T_S = 18$ $\quad N_S = 800$ rpm

$T_P = 24$ $\quad \eta = 0.94$

$T_C = 12$ $\quad N_A = 0$ in first case

$T_A = 72$ $\quad = 100$ rpm in second case

$P = 5$ kW

Prepare the Table 11.10:

Given conditions (1st case)

$$N_A = y - \frac{x}{8} = 0 \quad \text{or} \quad y = \frac{x}{8}$$

$\therefore$ $$N_S = y + x = \frac{x}{8} + x = 800$$

or $$\frac{9}{8}x = 800$$

or $$x = 711$$

$\therefore$ $$y = 88.9$$

$\therefore$ Speed of $a = y = \underline{88.9 \text{ rpm}}$

Now

$$\Sigma T\,N = 0$$

or $$\mathbf{T}_S N_S + \mathbf{T}_a N_a + \mathbf{T}_A N_A = 0$$

where

$$T_S = \frac{\text{Power input}}{\frac{2\pi N}{60}} = \frac{5000}{\frac{2\pi \times 800}{60}} = 59.7 \text{ N.m}$$

T_a = Theoretical torque at output

$$= \frac{\text{Actual torque at output}}{\text{Efficiency of power transmission}}$$

$$= \frac{T_a'}{\eta}$$

$\therefore$ $$59.7 \times 800 + \frac{T_a'}{0.94} \times 88.9 + T_A \times 0 = 0$$

or $$T_a' = -504.9 \text{ N.m}$$

Also,

$$T_S + T_a' + T_A = 0 \qquad \text{[Eq. (11.8)]}$$

or $$59.7 - 504.9 + T_A = 0$$

or $$T_A = \underline{445.2 \text{ N.m}}$$

2nd case:

$$N_A = y - \frac{x}{8} = 100 \text{ or } y = 100 + \frac{x}{8}$$

$$N_S = y + x = 100 + \frac{x}{8} + x = 800$$

or $$\frac{9x}{8} = 700$$

or $$x = 622.2$$

$$y = 100 + \frac{622.2}{8} = 177.8$$

$\therefore$ New speed of arm $a = y = \underline{177.8 \text{ rpm}}$

Table 11.10

Action	*a*	*S*	*P/C*	*A*
'a' is fixed, S + 1 rev.	0	1	$-\frac{18}{24}$	$-\frac{18}{24} \times \frac{12}{72}$
'a' is fixed, $S + x$ rev.	0	x	$-\frac{3x}{4}$	$-\frac{x}{8}$
Add y	y	$y + x$	$y - \frac{3x}{4}$	$y - \frac{x}{8}$

***Example* 11.12** *In the epicyclic gear train shown in Fig. 11.18, a gear C which has teeth cut internally and externally is free to rotate on an arm driven by the shaft S_1. It meshes externally with the casing D and internally with the pinion B. The gears have the following number of teeth:*
$T_B = 24$, $T_C = 32$ and 40, $T_D = 48$
Find the velocity ratio between
(i) S_1 and S_2 when D is fixed
(ii) S_1 and D when S_2 is fixed
What will be the torque required to fix the casing D if a torque of 300 N.m is applied to the shaft S_1?

Solution Complete the Table 11.11.

(i) From the given conditions,

$$N_D = y + \frac{5x}{8} = 0 \text{ or } y = -\frac{5x}{8}$$

$$\frac{N_{S1}}{N_{S2}} = \frac{N_a}{N_B} = \frac{y}{y+x} = \frac{-5x/8}{-\frac{5x}{8}+x} = -\frac{5}{3}$$

(ii) $$\frac{N_{S1}}{N_D} = \frac{N_a}{N_D} = \frac{y}{y+\frac{5x}{8}}$$

But $N_B = y + x = 0$ or $y = -x$

$$\therefore \frac{N_{S1}}{N_D} = \frac{-x}{-x+\frac{5x}{8}} = \frac{8}{3}$$

If T denotes the torque,

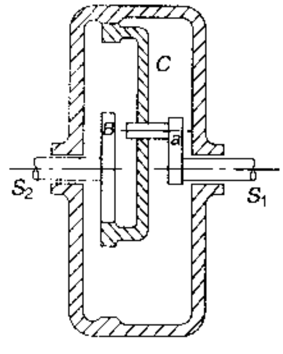


Fig. 11.18

$$\mathbf{T}_{S1}N_{S1} + \mathbf{T}_{S2}N_{S2} + \mathbf{T}_D N_D = 0$$

or

$$T_{S2} = -\frac{T_{S1}N_{S1}}{N_{S2}} = \frac{300 \times 5}{3} = 500 \text{ N.m}$$

($N_D = 0$, as the casing is fixed and $N_{S1}/N_{S2} = -5/3$)

Also,

$$\mathbf{T}_{S1} + \mathbf{T}_{S2} + \mathbf{T}_D = 0$$
$$300 + 500 + T_D = 0$$
or $$T_D = -800 \text{ N.m}$$

***Example* 11.13** *Figure 11.19 shows an epicyclic gear train in which the driving gear A has 20 teeth, the fixed annular gear C has 150 teeth and the ratio of teeth in gears D and E is 21:50. If 2 kW of power at a speed of 800 rpm is supplied to the gear A, determine the speed and the direction of rotation of gear E. Also, find the fixing torque required at the gear C.*

Table 11.11

Action	*a/S_1*	*B/S_2*	*C*	*D*
'*a*' fixed, *B* + 1 rev.	0	1	$\frac{24}{32}$	$\frac{24}{32} \times \frac{40}{48}$
'*a*' fixed, *B* + *x* rev.	0	x	$\frac{3x}{4}$	$\frac{5x}{8}$
Add *y*	y	$y + x$	$y + \frac{3x}{4}$	$y + \frac{5x}{8}$

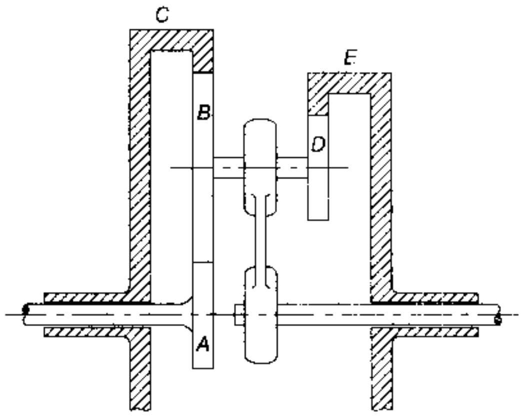


Fig. 11.19

Solution $T_A = 20$, $T_C = 150$, $N_D:N_E = 21:50$

Now, $T_C = 2\left[\frac{T_A}{2} + T_B\right]$

or $150 = 2\left[\frac{20}{2} + T_B\right]$ or $T_B = 65$

Prepare the Table 11.12:

For given conditions,

Gear A rotates at 800 rpm clockwise, $y + x = 800$ or $y = 800 - x$

Gear C is fixed, thus $y - \frac{2x}{15} = 0$

or $800 - x - \frac{2x}{15} = 0$ or $x = 705.9$

$y = 800 - 705.9 = 94.1$ rpm

Speed of $E = y - \frac{42x}{325}$

$= 94.1 - \frac{42 \times 705.9}{325} = 2.88$ rpm in the same direction as A.

If **T** denotes the torque,

$$\mathbf{T} = \frac{P}{\omega} = \frac{2000}{2\pi \times 800/60} = 23.87 \text{ N.m}$$

If there is no power loss, $\mathbf{T}_A N_A + \mathbf{T}_E N_E + \mathbf{T}_C N_C = 0$

$$\text{or } T_E = -\frac{T_A N_A}{N_E} = -\frac{23.87 \times 800}{2.88} = -6631 \text{ N.m}$$

... ($N_C = 0$, as the casing is fixed)

Also,

$$\mathbf{T}_A + \mathbf{T}_E + \mathbf{T}_C = 0$$

$$23.87 - 6631 + T_E = 0$$

or $T_E = 6607.13$ N.m

Table 11.12

Action	*a*	*A*	*B/D*	*C*	*E*
'a' fixed, A + 1 rev.	0	1	$-\frac{20}{65}$	$-\frac{20}{65} \times \frac{65}{150}$	$-\frac{20}{65} \cdot \left(\frac{21}{50}\right)$
'a' fixed, A + x rev.	0	x	$-\frac{4x}{13}$	$-\frac{2x}{15}$	$-\frac{42x}{325}$
Add y	y	$y + x$	$y - \frac{4x}{13}$	$y - \frac{2x}{15}$	$y - \frac{42x}{325}$

11.7 SUN AND PLANET GEAR

When an annular wheel A is added to the epicyclic gear train of Fig. 11.5, the combination is usually referred as *sun and planet gear* (Fig. 11.20). The annular wheel gears with the wheel P which can rotate freely on the arm a. The wheels S and P are generally called the *sun* and the *planet* wheels respectively due to analogy of motion of a planet around the sun.

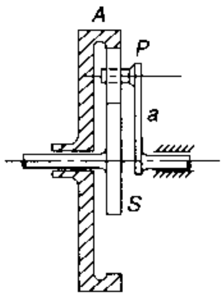


Fig. 11.20

In general, S, A and a are free to rotate independently of each other. It is also possible that either S or A are fixed. If A is fixed, S will be the driving member and if S is fixed, A will be the driving member. In each case the driven member is the arm a.

Let N_S = speed of the sun wheel S
N_A = speed of the annular wheel A
N_a = speed of the arm a
T_S = number of teeth on S
T_A = number of teeth on A

Then the Table 11.13 can be prepared as usual.

Table 11.13

Action	*Arm a*	*S*	*P*	*A*
'a' fixed, S + 1 rev.	0	1	$-\frac{T_S}{T_P}$	$\left(-\frac{T_S}{T_P}\times\frac{T_P}{T_A}\right)=-\frac{T_S}{T_A}$
'a' fixed, S + x rev.	0	x	$-\frac{T_S}{T_P}x$	$-\frac{T_S}{T_A}x$
All given y rev. (add y)	y	$y+x$	$y-\frac{T_S}{T_P}x$	$y-\frac{T_S}{T_A}x$

$\therefore$
$$x + y = N_S \quad \text{(i)}$$

and
$$y - \frac{T_S}{T_A}x = N_A \quad \text{(ii)}$$

Subtracting (ii) from (i),

$$x\left(1+\frac{T_S}{T_A}\right) = N_S - N_A$$

or
$$x\left(\frac{T_A+T_S}{T_A}\right) = N_S - N_A$$

or
$$x = \left(\frac{N_S-N_A}{T_A+T_S}\right)T_A$$

and
$$y = N_S - x$$

$$= N_S - \frac{N_ST_A - N_AT_A}{T_A+T_S}$$

$$= \frac{N_ST_A + N_ST_S - N_ST_A + N_AT_A}{T_A+T_S}$$

$$N_a = y = \frac{N_S T_S + N_A T_A}{T_S + T_A} \qquad (11.11)$$

If the sun wheel S is fixed, $N_S = 0$

Speed of the arm,

$$N_a = \frac{N_A T_A}{T_S + T_A} \text{ or } \frac{N_a}{N_A} = \frac{1}{T_S / T_A + 1}$$

If the annular wheel A is fixed, $N_A = 0$.

Speed of the arm,

$$N_a = \frac{N_S T_S}{T_S + T_A} \text{ or } \frac{N_a}{N_S} = \frac{T_S / T_A}{1 + T_S / T_A}$$

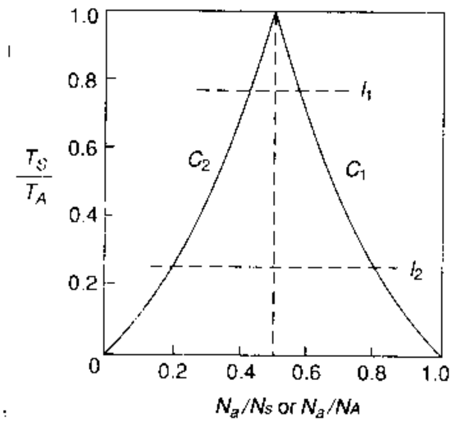


Fig. 11.21

The number of teeth on the sun wheel can vary from 0 to T_A, i.e., from zero to the number of teeth on the annular wheel. Therefore, the ratio of the number of teeth on the sun wheel to that on the annular wheel, i.e., T_S/T_A can vary from 0 to 1. If a graph T_S/T_A vs. N_a/N_A (S is fixed) is plotted, the curve C_1 is obtained (Fig. 11.21) which shows that the sun and the planet gear always acts as a reduction gear. The speed of the arm decreases from N_A to $1/2\ N_A$ as the number of teeth of the sun wheel increases from zero to T_A. Similarly, if A is fixed T_S/T_A vs. N_a/N_S is plotted, the curve C_2 is obtained. This shows that it again acts as a reduction gear in which the speed of the arm varies from zero to $1/2N_S$.

In both cases, the direction of the arm is the same as that of the driving member.

Horizontal dotted lines l_1 and l_2 show the practical limits of the ratio of T_S/T_A and of the corresponding speeds of the arm, where middle portion shows the range.

Example 11.14 *Determine the velocity ratio of the two shafts B and C of the compound gear shown in Fig. 11.22a in which the sun wheel S_2 is fixed. The numbers of teeth on different gears are mentioned alongside the respective gear. Also, find the torque required to fix the gear S_2 when a clockwise torque of 160 N.m is applied to the gear S_1.*

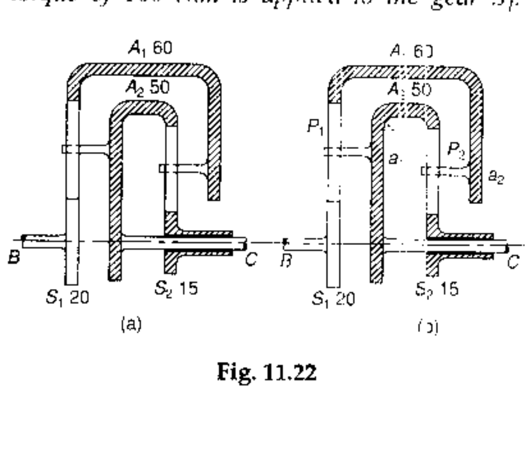


Fig. 11.22

Solution For the first sun and planet gear [Fig. 11.22(b)]

$$N_{a1} = \frac{N_{A1} T_{A1} + N_{S1} T_{S1}}{T_{A1} + T_{S1}}$$

$$= \frac{60 N_{A1} + 20 N_{S1}}{60 + 20}$$

$$= \frac{3 N_{A1} + N_{S1}}{4} \qquad (i)$$

For the second sun and planet gear (S_2 is fixed),

$$N_{a2} = \frac{50 N_{A2} + 0}{50 + 15} = \frac{10 N_{A2}}{13}$$

or $N_{a2} = 0.769\ N_{A2}$ (ii)

From the figure of the gear, it can be seen that

- Annular wheel A_1 is the arm a_2 of the second sun and planet gear
  Thus, $N_{A1} = N_{a2}$
- Annular wheel A_2 is the arm a_1 of the first sun and planet gear
  Thus, $N_{A2} = N_{a1}$
- Also $N_B = N_{S1}$ and $N_c = N_{A2}$

From (ii). $N_{A1} = 0.769 N_{A2}$

From (i), $N_{A2} = \dfrac{3N_{A1} + N_{S1}}{4}$

$= \dfrac{3 \times 0.769 N_{A2} + N_{S1}}{4}$

or $1.693\, N_{A2} = N_{S1}$

or $\dfrac{N_{S1}}{N_{A2}} = \dfrac{N_B}{N_C} = 1.693$

If **T** denotes the torque,

$\mathbf{T}_{S1} N_{S1} + \mathbf{T}_{S2} N_{S2} + \mathbf{T}_{A2} N_{A2} = 0$

or $\mathbf{T}_{S1} N_{S1} + 0 + \mathbf{T}_{A2} N_{A2} = 0$

or $\mathbf{T}_{A2} = -\dfrac{N_{S1}}{N_{A2}} \times T_{S1}$

$= -1.693 \times 160 = -270.9$ N.m

Also $\mathbf{T}_{S1} + \mathbf{T}_{S2} + \mathbf{T}_{A2} = 0$

or $160 + \mathbf{T}_{S2} - 270.9 = 0$

or torque required to fix the wheel S_2,

$\mathbf{T}_{S2} = 110.9$ N.m clockwise

Example 11.15 ***Figure 11.23 shows a compound gear in which an input torque of 150 N.m is given to shaft B at 1000 rpm. The sun and planet gears are all of the same diameter and pitch. What will be the speed and the torque at the output shaft C assuming an efficiency of 97%?***

Also, find the torque required to hold stationary the annulus A_1.

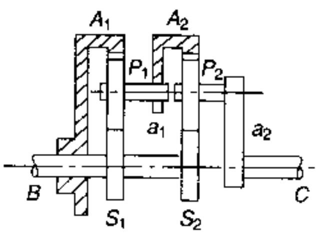


Fig. 11.23

Solution $N_{A2} = N_{a1}$, $N_{S1} = N_{S2} = 1000$ rpm

Speed of the arm a_1,

$$N_{a1} = \frac{N_{A1} T_{A1} + N_{S1} T_{S1}}{T_{A1} + T_{S1}} = \frac{N_{S1} T_{S1}}{T_{A1} + T_{A1}} \quad (N_{A1} = 0)$$

Similarly,

$$N_{a2} = \frac{N_{A2} T_{A2} + N_{S2} T_{S2}}{T_{A2} + T_{S2}}$$

All the sun and the planet gears are of the same diameter and pitch.

$$\therefore\ T_{A1} = 2\left[\frac{T_{S1}}{2} + T_{P1}\right] = 2\left[\frac{T_{S1}}{2} + T_{S1}\right] = 3T_{S1}$$

Thus,

$$T_{A1} = T_{A2} = 3T_{S1} = 3T_{S2} = 3T_{P1} = 3T_{P2}$$

Then $N_{a1} = \dfrac{N_{S1} T_{S1}}{T_{A1} + T_{S1}} = \dfrac{N_{S1} T_{S1}}{3T_{S1} + T_{S1}} = \dfrac{N_{S1}}{4}$

and $N_{a2} = \dfrac{3N_{A2} + N_{S2}}{4}$

$= \dfrac{3N_{a1} + N_{S1}}{4} \qquad (N_{A2} = N_{a1})$

$= \dfrac{3N_{S1}/4 + N_{S1}}{4}$

or $N_{a2} = \dfrac{7}{16} \times 1000 = 437.5$ rpm

Speed of shaft $C = 437.5$ rpm

If T denotes the torque,

$$T_S N_S + \frac{T_{a2} N_{a2}}{\eta} = 0$$

$$150 \times 1000 + \frac{T_{a2} \times 437.5}{0.97} = 0$$

$T_{a2} = \underline{-332.6 \text{ N.m}}$

Also $T_{S1} + T_{a2} + T_{A1} = 0$

$150 - 332.6 + T_{A1} = 0$

or

Holding torque, $T_{A1} = \underline{182.6 \text{ N.m}}$

in the same direction as the input torque.

11.8 BEVEL EPICYCLIC GEAR

The methods used for the solution of epicyclic trains of the spur gears are also valid for epicyclic trains consisting of bevel wheels. However, for wheels whose axes are inclined to the main axis, the terms clockwise and counter-clockwise are not applicable. So, in the tabular method of solution, plus or minus sign is omitted for these wheels and also the addition of y is not convenient to make as the rotation is about a different axis.

Example 11.16 *Figure 11.24 shows a Humpage gear used in a lathe headstock. The number of teeth on the wheels B, C, D, E and F are 18, 54, 22, 44 and 72 respectively. If the shaft G rotates at 200 rpm, what will be the speed of the shaft H when (i) F is held stationary, and (ii) it is rotated at 30 rpm opposite to G?*

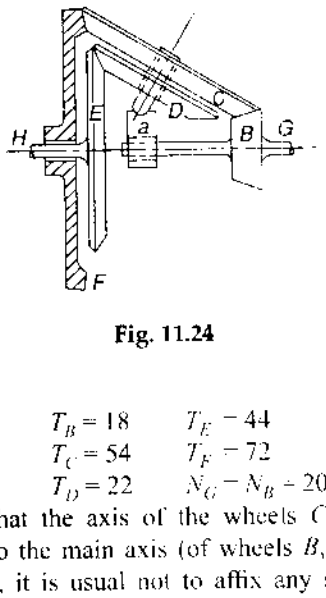


Fig. 11.24

Solution

$$T_B = 18 \qquad T_E = 44$$
$$T_C = 54 \qquad T_F = 72$$
$$T_D = 22 \qquad N_G = N_B = 200 \text{ rpm}$$

Note that the axis of the wheels C and D is inclined to the main axis (of wheels B, E and F). Therefore, it is usual not to affix any sign to the direction of rotation of these wheels as mentioned earlier. However, they influence the direction of rotation of the other wheels.

Prepare the Table 11.14.

From given conditions,

$$N_G = y + x = 200 \quad \text{or} \quad y = 200 - x$$

and $N_F = y - \dfrac{x}{4} = 0$

or $200 - x - \dfrac{x}{4} = 0$

or $x = 160$

$y = 200 - 160 = 40$

$\therefore \quad N_H = y - \dfrac{x}{6} = 40 - \dfrac{160}{6} = \underline{13.3 \text{ rpm}}$

In the second case,

$$N_G = y + x = 200 \quad \text{or} \quad y = 200 - x$$

and $N_F = y - \dfrac{x}{4} = -30$

or $200 - x - \dfrac{x}{4} = -30$

or $x = 184$

$y = 200 - 184 = 16$

$\therefore \quad N_A = y - \dfrac{x}{6} = 16 - \dfrac{184}{6} = \underline{-14.67 \text{ rpm}}$

Thus, in the second case, the shaft H rotates in the opposite direction to that of G.

Example 11.17 *Figure 11.25 shows a train of bevel gears. The wheel E is fixed whereas the wheels B and G are keyed to the driving and the driven shafts respectively. The wheels C, D and F are keyed to the inclined shaft which is supported on the arm a. The arm is free to rotate about the common axis of the driving and driven shafts. The number of teeth on the wheels B, C, D, E, F and G are 15, 45, 45, 135, 40 and 100 respectively. Find the ratio of the driving and the driven shaft speeds.*

Table 11.14

Action	*a*	*B/G*	*C/D*	*E/H*	*F*
'*a*' fixed, *B* +1 rev.	0	1	$\dfrac{18}{54}$	$-\dfrac{18}{54} \times \dfrac{22}{44}$	$-\dfrac{18}{54} \times \dfrac{54}{72}$
'*a*' fixed, *B* + *x* rev.	0	x	$\dfrac{x}{3}$	$-\dfrac{x}{6}$	$-\dfrac{x}{4}$
Add *y*	y	$y + x$	$y + \dfrac{x}{3}$	$y - \dfrac{x}{6}$	$y - \dfrac{x}{4}$

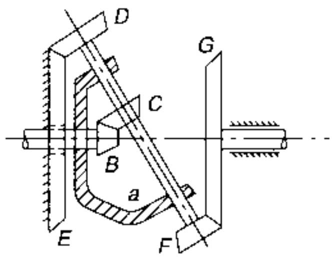


Fig. 11.25

Solution

$T_B = 15$ $T_E = 135$

$T_C = 45$ $T_F = 40$

$T_D = 45$ $N_G = 100$

From the figure, it is clear that the train consists of two epicyclic trains. The train *BCDE* causes the arm *a* to turn and the arm causes *G* to turn through the train *EDFG*.

Prepare Table 11.15.

From the given conditions,

$$N_E = y + \frac{x}{9} = 0 \quad \text{or} \quad x = -9y$$

$$\frac{N_G}{N_B} = \frac{y + \frac{2x}{15}}{y + x} = \frac{y - \frac{18y}{15}}{y - 9x} = -\frac{-\frac{y}{5}}{-8y} = \frac{1}{40}$$

Thus, *B* must turn 40 times to turn *G* once in the same direction.

Table 11.15

Action	a	B/G	C/D/F	E	G
'*a*' fixed, *B* + 1 rev.	0	1	$\frac{15}{45}$	$\frac{15}{45} \times \frac{45}{135}$	$\frac{15}{45} \times \frac{40}{100}$
'*a*' fixed, *B* + *x* rev.	0	*x*	$\frac{x}{3}$	$\frac{x}{9}$	$\frac{2x}{15}$
Add *y*	*y*	$y + x$	$y + \frac{x}{3}$	$y + \frac{x}{9}$	$y + \frac{2x}{15}$

11.9 COMPOUND EPICYCLIC GEAR

When an epicyclic gear consists of a number of epicyclic gears (sun and planet gears) in series such that the pin of the arm of the first epicyclic gear drives an element of another epicyclic gear, it is known as a *compound epicyclic gear.*

Figure 11.26 shows a compound epicyclic gear. It consists of three simple epicyclic gears namely $A_1a_1P_1S_1$, $A_2a_2P_2S_2$ and $A_3a_3P_3S_3$. The planet P_1 of the first epicyclic gear rotates freely on the pin carried by the arm a_1. As the arm a_1 is integral with the annulus A_2 of the second epicyclic gear and the sun wheel of the third, the pin of the arm a_1 also drives A_2 and S_3, i.e., the annulus and the sun wheel of the second and the third epicyclic gears respectively at the same speed and direction as its own about the axis of the arm. Also, the sun wheel of the first is integral with that of second and the arm of the second with that of the third.

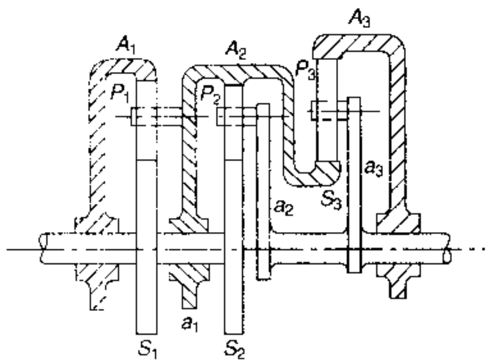


Fig. 11.26

To analyse a compound epicyclic gear, each epicyclic gear or sun and planet gear is treated separately. Thus, for the first epicyclic gear,

Speed of the arm,

$$N_{a1} = \frac{N_{S1}T_{S1} + N_{A1}T_{A1}}{T_{S1} + T_{A1}} \tag{11.12}$$

Thus, if the speeds N_{S1} and N_{A1} are known (if A_1 is fixed, $N_{A1} = 0$), N_{a1} can be calculated.

Similarly, for the second sun and planet gear,

$$N_{a2} = \frac{N_{S2}T_{S2} + N_{A2}T_{A2}}{T_{S2} + T_{A2}}$$

As A_2 is integral with a_1 and S_2 with S_1,

$$\therefore \qquad N_{A2} = N_{a1} \qquad \text{and} \qquad N_{S2} = N_{S1}$$

Thus, N_{a2} can be known.

In the same way,

$$N_{a3} = \frac{N_{S3}T_{S3} + N_{A3}T_{A3}}{T_{S3} + T_{A3}}$$

a_3 is integral with a_2 and S_3 with a_1,

$$\therefore \qquad N_{a3} = N_{a2} \qquad \text{and} \qquad N_{S3} = N_{a1}$$

So, N_{A3} can be calculated.

11.10 AUTOMOTIVE TRANSMISSION GEAR TRAINS

Gear trains may be used to obtain different speeds of an automobile. A simple sliding gear box makes use of a compound gear train and is engaged by sliding the gears on the driven shaft to mesh with the gears on a lay shaft. On the other hand, a pre-selective gear box uses sun and planet gears and brake bands are used to lock one of the annular wheels.

1. Sliding Gear Box

The arrangement of such type of a gear box is shown in Fig. 11.27. The pinion *A*, keyed to the driving shaft is in constant mesh with the gear *B* on the lay shaft. The gears *B*, *C*, *E* and *G* are rigidly fixed on the lay shaft. The driven shaft is splined and carries the gear *D* as well as the compound gear *F–H*. Thus, gears *D*, *F* and *H* revolve with the driven shaft and can also slide on it. The figure shows the gear box in the neutral position, i.e., if the driving shaft is revolving, all the gears on the lay shaft revolve, but the driven shaft with its gears will be at rest.

First Gear The first gear is engaged by the sliding gear H towards the right and meshing it with the gear G of the lay shaft. The transmission will be from A to B and from G to H and the train value $\frac{T_A}{T_B} \cdot \frac{T_G}{T_H}$.

Second Gear The vehicle is engaged in the second gear by the sliding gear F towards left and engaging it with the gear E of the lay shaft. The transmission is from A to B and from E to F and the train value $\frac{T_A}{T_B} \cdot \frac{T_E}{T_F}$.

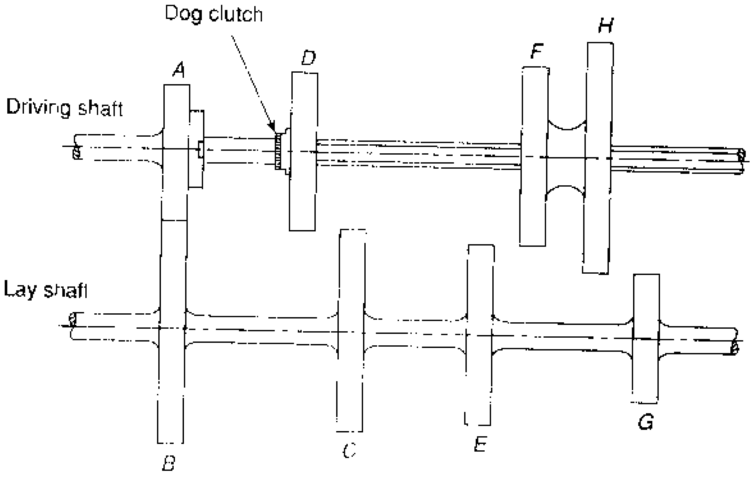


Fig. 11.27

Third Gear By sliding the gear D towards the right and engaging with the gear C of the lay shaft, the transmission in the third gear is obtained which is from A to B and from C to D and the train value $\frac{T_A}{T_B} \cdot \frac{T_C}{T_D}$.

Top Gear The gear D is engaged directly with the gear A through a dog clutch. This way the power is transmitted directly to the driven shaft. The lay shaft along with its gear wheels revolves idly and the driven shaft runs at the same speed as the driving shaft.

To put the vehicle in the reverse gear, an idler is made to mesh with G and H (not shown in the figure) so that both of them rotate in the same direction, thus rotating the driven shaft in the opposite direction.

2. Pre-Selective Gear Box

A pre-selective gear-box is a device by which three or four different speeds of the automobile can be obtained. It makes use of sun and planet gears. The number of sun and planet gears to be used will be equal to the number of gear ratios required, except for the top gear which is obtained by direct drive from the crankshaft to the propeller shaft through the clutch.

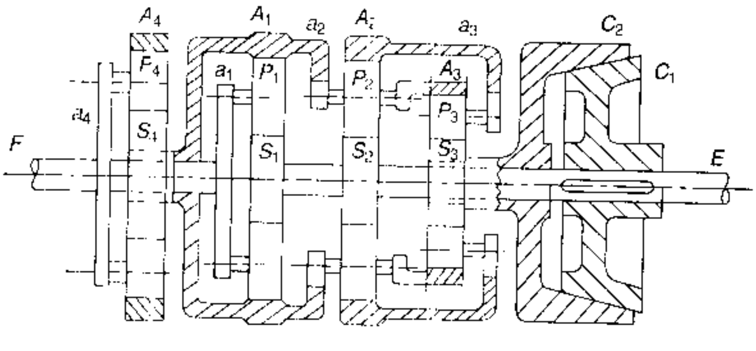


Fig. 11.28

A typical pre-selective gear-box known as *Wilson gear box* is shown in Fig. 11.28. It consists of four sets of the sun and planet gears out of which the first three are for speed reduction and the fourth, for the reverse gear. The first set of the sun and planet gear gives the minimum speed of the propeller shaft whereas the third, the maximum, but lesser than that obtained with the top gear.

The engine shaft E is made integral with the sun wheels S_1 and S_2 and is keyed to the inner element C_1 of the clutch. The sun wheel S_3 is integral with the outer element C_2 of the clutch and can rotate freely around the shaft E.

The arm a_3, carrying the pin around which the planet P_3 can rotate freely, is integral with the annular wheel A_2. The arm a_2, carrying the pin around which the planet P_2 can rotate freely is integral with the annular wheels A_1 and A_3. The arm a_1 carrying the pin for the planet P_1 is integral with the propeller shaft F and the arm a_4. The arm a_4 carries a pin for the planet P_4.

The sun wheel S_4 is integral with the annular wheel A_1 and can rotate freely around the shaft F.

All the sun wheels gear with their annular wheels through their respective planet gears.

Brake bands (not shown in the figure) are provided around the annular wheels A_1, A_2, A_4 and the element C_2 of the clutch by which any of these can be locked in turn. During the locking of any of these, the two elements C_1 and C_2 of the clutch are disengaged automatically. The clutch is engaged only for the direct drive (top gear).

First Gear To engage the first gear, A_1 is locked and the clutch is disengaged. Consider the first set of the sun and planet gear,

Speed of arm,

$$N_{a1} = \frac{N_{A1}T_{A1} + N_{S1}T_{S1}}{T_{A1} + T_{S1}}$$

where N_{S1} = speed of the sun wheel S_1
= speed of the engine shaft E
N_{A1} = speed of the annular wheel A_1
= 0

Therefore, N_{a1} can be known which is also the speed of the shaft F.

The speed of the other gears which rotate freely can also be known as follows:

For the second set of the sun and planet gear,

$$N_{a2} = \frac{N_{A2}T_{A2} + N_{S2}T_{S2}}{T_{A2} + T_{S2}}$$

where $N_{a2} = N_{A1} = 0$
and $N_{S2} = N_{S1}$ = speed of E
Thus, N_{A2} can be known.

$$N_{a3} = \frac{N_{A3}T_{A3} + N_{S3}T_{S3}}{T_{A3} + T_{S3}}$$

$N_{A3} = N_{a2} = 0$
and $N_{a3} = N_{A2}$ (calculated above)

N_{S3} can be calculated. S_3 is integral with C_2 and thus C_2 also rotates at the same speed and has free rotation.

$$N_{a4} = \frac{N_{A4}T_{A4} + N_{S4}T_{S4}}{T_{A4} + T_{S4}}$$

where $N_{a4} = N_{a1}$
and $N_{S4} = N_{A1} = 0$

Thus, N_{A4} can be calculated. The annual wheel A_4 has free rotation.

Second Gear To engage the second gear, A_2 is locked and the clutch is disengaged.

$$N_{a2} = \frac{N_{A2}T_{A2} + N_{S2}T_{S2}}{T_{A2} + T_{S2}}$$

where $N_{A2} = 0$

and $N_{S2} =$ Speed of E

Thus, N_{a2} can be known.

$$N_{a1} = \frac{N_{A1}T_{A1} + N_{S1}T_{S1}}{T_{A1} + T_{S1}}$$

where $N_{A1} = N_{a2}$

and $N_{S1} = N_E$

N_{a1} which is also the speed of the propeller shaft, can be known.

Free rotation of A_4 and C_2 can also be known in a similar way as in case of the first gear.

Third Gear Lock C_2 and disengage the clutch. The operation also locks the sun wheel S_3.

$$N_{a3} = \frac{N_{A3}T_{A3} + N_{S3}T_{S3}}{T_{A3} + T_{S3}} = \frac{N_{A3}T_{A3}}{T_{A3} + T_{S3}} \qquad (N_{S3} = 0)$$

and

$$N_{a2} = \frac{N_{A2}T_{A2} + N_{S2}T_{S2}}{T_{A2} + T_{S2}}$$

In the above two equations,

$$N_{a3} = N_{A2},\ N_{a2} = N_{A3} \text{ and } N_{S2} = N_E$$

Thus, N_{a2} can be calculated. From N_{a2} the value of N_{a1} can be calculated in the way it is done for the second gear which further provides the speed of the propeller shaft.

Fourth (Top) Gear All the brake bands are freed. C_1 and C_2 are engaged. Thus, S_1, S_2 and S_3 all rotate at the engine speed and it can be shown that the whole system rotates as a complete unit at the engine speed. Thus, speed of the propeller shaft is equal to that of the engine shaft.

Reverse Gear The clutch is disengaged and A_4 is locked.

We have,

$$N_{a4} = \frac{N_{A4}T_{A4} + N_{S4}T_{S4}}{T_{A4} + T_{S4}}$$

$$= \frac{N_{S4}T_{S4}}{T_{A4} + T_{S4}} \qquad (N_{A4} = 0)$$

and

$$N_{a1} = \frac{N_{A1}T_{A1} + N_{S1}T_{S1}}{T_{A1} + T_{S1}}$$

In the above two equations,

$N_{a4} = N_{a1}$,

$N_{S4} = N_{A1}$

and $N_{S1} = N_E$

Thus, N_{a1} or N_{a4} can be known which is the speed of the propeller shaft.

Assuming roughly that $T_{A1} = T_{A4}$ and $T_{S1} = T_{S4}$,

$$N_{S4}T_{S4} = N_{A1}T_{A1} + N_{S1}T_{S1} = N_{S4}T_{A1} - N_{S1}T_{S1}$$
$$N_{S4}(T_{S4} - T_{A1}) = N_{S1}T_{S1}$$
$$N_{S4} = \frac{N_{S1}.T_{S1}}{T_{S4} - T_{A1}}$$

As T_{A1} is greater than T_{S4}, N_{S4} will be negative. Thus, N_{S4} is negative, which shows that the propeller shaft rotates in the reverse direction.

Example 11.18 *A four-speed sliding gear box of an automobile is to be designed to give speed ratios of 4, 2.5, 1.5 and 1 approximately for the first, second, third and top gears respectively. The input and the output shafts have the same alignment. The horizontal central distance between them and the lay shaft is 90 mm. The teeth have a module of 4 mm. No wheel has less than 15 teeth. Calculate suitable number of teeth on each wheel and find the actual speed ratios attained.*

Solution

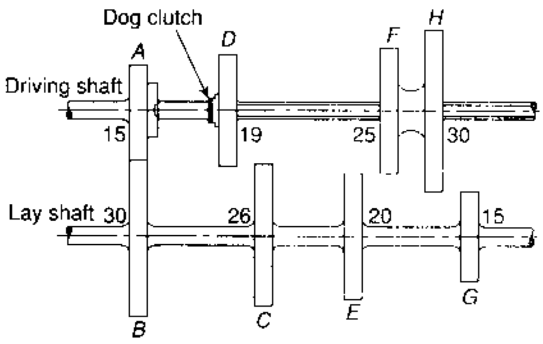


Fig. 11.29

First Gear (Fig. 11.29) The transmission is from A to B and from G to H.

As two shafts are parallel, $r_a + r_b = r_g + r_h$

$$= \frac{mT_a}{2} + \frac{mT_b}{2} = \frac{mT_g}{2} + \frac{mT_h}{2} = 90$$

$$T_a + T_b = T_g + T_h = \frac{90 \times 2}{4} = 45 \qquad \text{(i)}$$

$$\text{The train value} = \frac{T_a}{T_b} \cdot \frac{T_g}{T_h} = \frac{1}{4}$$

To achieve this train value, the ratio of number of teeth between A and B, and G and H may be assumed same.

Thus, $T_b = 2T_a$ and $T_h = 2T_g$. (ii)

From (i) and (ii), $3T_a = 45$ or $T_a = 15$, $T_b = 30$

Similarly, $T_g = 15$, $T_h = 30$

Second Gear The transmission is from A to B and from E to F.

$$T_e + T_f = T_a + T_b = 45 \qquad \text{(iii)}$$

$$\text{The train value} = \frac{T_a}{T_b} \cdot \frac{T_e}{T_f} = \frac{1}{2.5} \text{ or } \frac{15}{30} \cdot \frac{T_e}{T_f} = \frac{1}{2.5}$$

$$\text{or} \quad \frac{T_e}{T_f} = 0.8 \qquad \text{(iv)}$$

From (iii) and (iv), $1.8T_f = 45$ or $T_f = 25$

and $T_e = 45 - 25 = 20$

$$\text{Speed ratio} = \frac{T_b}{T_a} \cdot \frac{T_f}{T_e} = \frac{30}{15} \cdot \frac{25}{20} = 2.5$$

Third Gear The transmission is from A to B and from C to D.

$$T_c + T_d = T_a + T_b = 45 \qquad \text{(v)}$$

$$\text{The train value} = \frac{T_a}{T_b} \cdot \frac{T_c}{T_d} = \frac{1}{1.5} \text{ or } \frac{15}{30} \cdot \frac{T_c}{T_d} = \frac{1}{1.5}$$

$$\text{or} \quad \frac{T_c}{T_d} = 1.333 \qquad \text{(vi)}$$

From (v) and (vi), $2.333\ T_d = 45$ or $T_d = 19.3$ say 19 teeth

and $T_c = 45 - 19 = 26$

$$\text{Speed ratio} = \frac{T_b}{T_a} \cdot \frac{T_d}{T_c} = \frac{30}{15} \cdot \frac{19}{26} = 1.46$$

Top Gear Gear D is engaged directly with the gear A through a dog clutch to obtain a speed ratio of 1. This way the power is transmitted directly to the driven shaft and the driven shaft runs at the same speed as the driving shaft.

Example 11.19 *In the pre-selective gear-box shown in Fig. 11.28, the number of teeth are:*

$T_{A1} = T_{A2} = 72 \quad T_{S1} = T_{S2} = 22$

$T_{A3} = 63 \quad T_{S3} = 19$

$T_{A4} = 81 \quad T_{S4} = 37$

If the input shaft E rotates at a uniform speed of 800 rpm, determine the speeds of the output shaft F when different gears are engaged.

Solution *When the first gear is engaged*

$$N_{a1} = \frac{N_{S1}T_{S1}}{T_{A1} + T_{S1}} = \frac{800 \times 22}{72 + 22} = 187.2$$

or $N_F = \underline{187.2 \text{ rpm}}$

When the second gear is engaged

$$N_{a2} = \frac{N_{S2}T_{S2}}{T_{A2} + T_{S2}} = \frac{800 \times 22}{72 + 22} = 187.2$$

$$N_{a1} = \frac{N_{A1}T_{A1} + N_{S1}T_{S1}}{T_{A1} + T_{S1}} = \frac{N_{a2}T_{A1} + N_{S1}T_{S1}}{T_{A1} + T_{S1}}$$

$$= \frac{187.2 \times 72 + 800 \times 22}{72 + 22}$$

or $N_F = N_{a1} = \underline{330.6 \text{ rpm}}$

When the third gear is engaged

$$N_{a3} = \frac{N_{A3}T_{S3}}{T_{A3} + T_{S3}} = \frac{63N_{A3}}{63 + 19} = \frac{63N_{A3}}{82}$$

or

$$N_{A3} = \frac{82N_{a3}}{63}$$

$$N_{a2} = \frac{N_{A2}T_{A2} + N_{S2}T_{S2}}{T_{A2} + T_{S2}}$$

$$= \frac{N_{a3}T_{A2} + N_{S2}T_{S2}}{T_{A2} + T_{S2}} \qquad (N_{A2} = N_{a3})$$

$$= \frac{\frac{63N_{A3}}{82}T_{A2} + N_{S2}T_{S2}}{T_{A2} + T_{S2}}$$

$$= \frac{\frac{63N_{a2}}{82}T_{A2} + N_{S2}T_{S2}}{T_{A2} + T_{S2}} \qquad (N_{a2} = N_{A3})$$

$$= \frac{\frac{63N_{a2}}{82} \times 72 + 800 \times 22}{72 + 22}$$

or $94N_{a2} = 55.32N_{a2} + 17\,600$

or $38.68\,N_{a2} = 17\,600$

or $N_{a2} = 455 = N_{A1}$

$$N_{A1} = \frac{N_{A1}T_{A1} + N_{S1}T_{S1}}{T_{A1} + T_{S1}} = \frac{455 \times 72 + 800 \times 22}{94}$$

or $N_F = N_{a1} = \underline{535.7 \text{ rpm}}$

When the fourth gear is engaged

$N_F = N_E = \underline{800 \text{ rpm}}$

In the reverse gear

$$N_{a4} = \frac{N_{S4}T_{S4}}{T_{A4} + T_{S4}} \qquad (N_{A4} = 0)$$

or

$$N_{a4} = \frac{37N_{S4}}{81 + 37} = \frac{37N_{S4}}{118}$$

$$N_{a1} = \frac{N_{A1} \times 72 + 800 \times 22}{94}$$

But

$N_{a4} = N_{a1}$ and $N_{S4} = N_{A1}$

$$\frac{37N_{S4}}{118} = \frac{72N_{S4} + 17\,600}{94}$$

or $0.452\,N_{S4} = -187.23$

or $N_{S4} = -413.9$

$$N_F = N_{a4} = -\frac{37 \times 413.9}{118} = \underline{-129.8 \text{ rpm}}$$

11.11 DIFFERENTIALS

Differential means to differentiate which may be between two speeds or two values or two readings. Differentials are usually two-degree-of-freedom mechanisms in which two inputs or coordinates must be defined to obtain a definite output.

Automotive Differential

When a vehicle takes a turn, the outer wheels must travel farther than the inner wheels. In automobiles, the front wheels can rotate freely on their axes and thus can adapt themselves to the conditions. However, both rear wheels are driven by the engine through gearing. Therefore, some sort of automatic device is necessary so that the two rear wheels are driven at slightly different speeds. This is accomplished by fitting a *differential gear* on the rear axle.

The fact that an epicyclic gear has two degrees of freedom has been utilised in the differential gear of an automobile. It permits the two wheels to rotate at the same speed when driving in straight while allowing the wheels to rotate at different speeds when taking a turn. Thus, a differential gear is a device which adds or subtracts angular displacements.

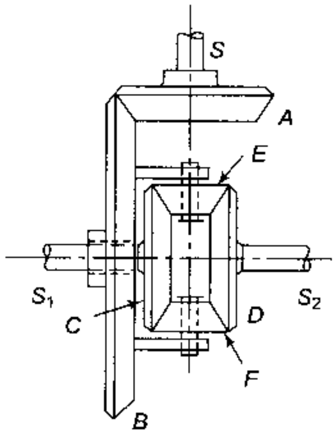


Fig. 11.30

Differential of an automobile

Figure 11.30 shows the arrangement of gears in the differential of an automobile. The shaft S is driven by the engine through the gear-box and has a bevel pinion A keyed to it. The bevel pinion A meshes with a bevel wheel B which turns loosely on the hub of the gear C. Shafts S_1 and S_2 form the rear axle to which are fixed the rear wheels of the automobile. Gears C and D are keyed to the shafts S_1 and S_2 respectively. C and D gear with equal bevel pinions E and F which are free to rotate on their respective axes. The wheel B carries two brackets that support the bearings for gear S E and F.

When the automobile moves in a straight path, the bevel pinion A drives the wheel B. The whole differential acts as one unit and rotates with the bevel wheel B so that the wheels C and D rotate with the same speed and in the same direction as B. There is no relative motion between gears C and D, and E and F. Gears E and F also do not rotate about their own axes. They act just like keys to transmit motion from B to C and D. Thus, C and D, which are keyed to the shafts that carry the wheels, rotate at the same speed as B.

When a turn is taken, E and F rotate about their own axes and the system works as an epicyclic gear giving two outputs at C and D with one input at B. Prepare a table as below:

Action	*B*	*C/S₁*	*D/S₂*	*E/F*
B fixed, C + 1 rev.	0	1	−1	$\frac{T_C}{T_E}$
B fixed, C + x rev.	0	x	$-x$	$\frac{T_C}{T_E}x$
Add y	y	$y+x$	$y-x$	$\frac{T_C}{T_E}x+y$

From the above table, it can be observed that the speed of B is the arithmetical mean of the speeds of C and D, because $y = \{(y + x) + (y - x)\}/2$. This shows that while taking a turn, if the speed of C decreases than that of B, there will be a corresponding increase in the speed of D.

Example 11.20 *In the differential gear of a car shown in Fig. 11.30 the number of teeth on the pinion A on the propeller shaft is 18 whereas the crown gear B has 90 teeth. If the propeller shaft rotates at 1200 rpm and the wheel attached to the shaft S_2 has a speed of 255 rpm, while the vehicle takes a turn, determine the speed of the wheel attached to the shaft S_1.*

Solution

Speed of the gear $B = N_A \times \frac{T_B}{T_A} = 1200 \times \frac{18}{90}$

$= 240$ rpm

Thus, $y = 240$

Prepare the table as in the above section.

Speed of $S_2 = y - x = 255$

or $240 - x = 255$

or $x = -15$

Speed of $S_1 = y + x = 240 - 15 = \underline{225 \text{ rpm}}$

Summary

1. A gear train is a combination of gears used to transmit motion from one shaft to another.
2. The main types of gear trains are *simple*, *compound*, *reverted* and *planetary* or *epicyclic*.
3. A series of gears, capable of receiving and transmitting motion from one gear to another is called a *simple gear train*. All the gear axes remain fixed relative to the frame and each gear is on a separate shaft.
4. When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity, it is known as a *compound gear train*.
5. If the axes of the first and the last wheels of a compound gear coincide, it is called a *reverted gear train*.
6. A gear train having a relative motion of axes is called a *planetary* or an *epicyclic gear train* (or simply epicyclic gear or train). Thus, in an epicyclic train, the axis of at least one of the gears also moves relative to the frame.
7. In general, gear trains have two degrees of freedom. However, the number of inputs can be reduced to one, if one wheel of the train is fixed. That amounts to reducing the speed of that gear wheel to zero.
8. Large speed reductions are possible with epicyclic gears and if the fixed wheel is annular, a more compact unit could be obtained.
9. When an annular wheel is added to the epicyclic gear train, the combination is usually referred as *sun and planet gear*.
10. When an epicyclic gear consists of a number of sun and planet gears in series such that the pin of the arm of the first drives an element of another, it is known as a *compound epicyclic gear*.
11. A simple sliding gear box makes use of a compound gear train and is engaged by sliding the gears on the driven shaft to mesh with the gears on a lay shaft.
12. A pre-selective gear-box is a device by which three or four different speeds of the automobile can be obtained. It makes use of sun and planet gears.
13. A differential of an automobile permits the two wheels of a vehicle to rotate at the same speed when driving in straight while allowing the wheels to rotate at different speeds when taking a turn. Thus, a differential gear is a device which adds or subtracts angular displacements.

Exercises

1. What is a gear train? What are its main types?
2. What is the difference between a simple gear train and a compound gear train? Explain with the help of sketches.
3. What is a reverted gear train? Where is it used?
4. Explain the procedure to analyse an epicyclic gear train.

5. What do you mean by braking or the fixing torque of a gear in an epicyclic gear train?
6. What is a sun and planet gear? Give the procedure to analyse such a gear train?
7. What do you mean by a compound epicyclic gear?
8. Sketch a sliding gear box and explain its working.
9. Describe the function of a pre-selective gear-box of an automobile.
10. What do you mean by differentials? Give examples.
11. What is a differential gear of an automobile? How does it function?
12. A compound train consists of four gears. The number of teeth on gears *A*, *B* , *C* and *D* are 54, 75, 36 and 81 respectively. Gears *B* and *C* constitute a compound gear. Determine the torque on the output shaft if the gear *A* transmits 9 kW at 200 rpm and the train efficiency is 80%.

(1.074 kN.m)

13. An epicyclic gear consists of a pinion, a wheel of 40 teeth and an annulus with 84 internal teeth concentric with the wheel. The pinion gears with the wheel and the annulus. The arm that carries the axis of the pinion rotates at 100 rpm. If the annulus is fixed, find the speed of the wheel; if wheel is fixed, find the speed of the annulus.

(310 rpm; 147.6 rpm)

14. In an epicyclic gear shown in Fig. 11.12, the pitch circle diameter of the annulus *A* is to be approximately 324 mm and the module is to be 6 mm. When the annulus is stationary, the three-armed spider makes one revolution for every five revolutions of the wheel *S*. Find the number of teeth for all the wheels and exact pitch circle diameter of the annulus. If a torque of 30 N.m is applied to the shaft carrying *S*, determine the fixing torque of the annulus.

($T_S = 14$, $T_P = 21$, $T_A = 56$; 120 N.m)

15. Determine a suitable train of wheels to satisfy the requirements of a clock, the minute hand of which is fixed to a spindle and the hour hand to a sleeve rotating freely on the same spindle. The pitch is the same for all the wheels and each wheel has at least 11 teeth. The total number of teeth should be as small as possible.

(12, 48 and 15, 45)

16. The pinion *S* (Fig. 11.13) has 15 teeth, and is rigidly fixed to a motor shaft. The wheel *P* has 20 teeth and gears with *S* and also with a fixed annulus wheel *A*. The pinion *C* has 15 teeth and is fixed to the wheel *P*. *C* gears with the annular wheel *D*, which is keyed to a machine shaft. *P* and *C* can rotate together on a pin carried by an arm which rotates about the shaft on which *S* is fixed. Find the speed of the machine shaft if the motor rotates at 1000 rpm.

(37.15 rpm in the same direction as *S*)

17. In an epicyclic gear (Fig. 11.12), the wheel *A* has also external teeth and is driven by a pinion *B*. The number of teeth on *S* = number of teeth on *B* = one fourth of number of teeth on *A*. The wheel *A* has same number of teeth internally and externally.

Determine the speed of the driven shaft fixed to the arm *a*, when *B* makes 600 rpm in the same direction as *S* whose speed is 400 rpm. Also, find the speed and direction of rotation of *B* if the driven shaft rotates at the same speed as above but in the opposite direction.

(40 rpm in a direction opposite to *S*;
200 rpm in direction of *S*)

18. Figure 11.31 shows an epicyclic train known as Ferguson's paradox. The gears have number of teeth as indicated. Gear 1 is fixed to the frame and is stationary. The arm *a* and the gears 2 and 3 are free to rotate on the shafts. The pitch circle diameters of all are the same so that the planet gear *P* meshes with them all. Find the number of revolutions of gears 2 and 3 for one revolution of arm *a*.

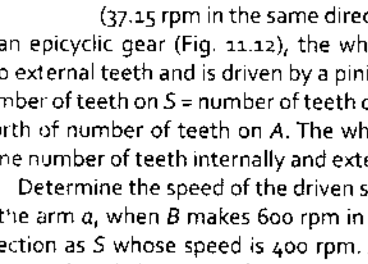


Fig. 11.31

(2 makes 1/81 revs. in the same direction and 3 makes 1/79 revs in the opposite direction of arm *a*)

Note: If the three gears 1, 2 and 3 have the same diameter, their pitches must be slightly different and theoretically, the drive will not be perfect.

19. In the epicyclic gear of Fig. 11.15, T_S = 40. T_P = 20 and T_A = 80. If the sun gear rotates at 150 rpm counter-clockwise and the annular ring clockwise at 400 rpm, find the arm speed.

(216.7 rpm clockwise)

20. In an epicyclic gear (Fig. 11.32), the wheel A fixed to S_1 has 30 teeth and rotates at 500 rpm. B gears with A and is fixed rigidly to C, both being free to rotate on S_2. The wheels B, C and D have 50, 70 and 90 teeth respectively. If D rotates at 80 rpm in a direction opposite to that of A, find the speed of the shaft S_2. (104.5 rpm in same direction)

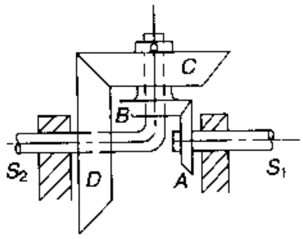


Fig. 11.32

21. In an indexing mechanism of a milling machine (Fig. 11.30), the drive is from gear wheels fixed to shafts S_1 and S_2 to the bevel A through the gear train. The number of teeth of A, B, D and F are 30, 60, 28 and 24 respectively. Each gear has a module of 10 mm.
Determine the number of revolutions of S (or A) for one revolution of S_1 when
(a) S_1 and S_2 have same speed in the same direction
(b) S_1 and S_2 have same speed in the opposite direction
(c) S_1 makes 48 rpm and S_2 is at rest
(d) S_1 makes 48 rpm and S_2 24 rpm in the same direction
(2; zero; 48 rpm; 72 rpm)

22. Show that in a Humpage reduction gear (Fig. 11.24), the wheel E rotates in the same direction as the wheel B if T_C/T_D is more than T_F/T_E and in the opposite direction if the same is less than T_F/T_E. Gear F is the fixed frame.

23. In a sun and planet gear train, the sun gear wheel having 60 teeth is fixed to the frame. Determine the numbers of teeth on the planet and the annulus wheels if the annulus rotates 130 times and the arm rotates 100 times, both in the same direction. (70; 200)

24. A four-speed sliding gear box of an automobile is to be designed to give approximate speed ratios of 4, 2.4, 1.4 and 1 for the first, second, third and top gears respectively. The input and the output shafts have the same alignment. Horizontal central distance between them and the lay shaft is 98 mm. The teeth have a module of 4 mm. No wheel has less than 16 teeth. Calculate suitable number of teeth on each wheel and find the actual speed ratios attained.

25. In the pre-selective gear-box shown in Fig. 11.28, the number of teeth are
$T_{A1} = T_{A2} = 80$ $T_{S1} = T_{S2} = 24$
$T_{A3} = 68$ $T_{S3} = 21$
$T_{A4} = 90$ $T_{S4} = 41$
If the input shaft E rotates at a uniform speed of 640 rpm, determine the speeds of the output shaft F when different gears are engaged.
(147.7 rpm, 261.3 rpm, 423 rpm, 640 rpm, 101.4 rpm)

26. In the differential gear of a car shown in Fig. 11.30, the number of teeth on the pinion A on the propeller shaft is 24 whereas the crown gear B has 128 teeth. If the propeller shaft rotates at 800 rpm and the wheel attached to the shaft S_2 has a speed of 175 rpm, determine the speed of the wheel attached to shaft S_1 when the vehicle takes a turn. (125 rpm)

12 STATIC FORCE ANALYSIS

Introduction

In all types of machinery, forces are transmitted from one component to the other such as from a belt to a pulley, from a brake drum to a brake shoe, from a gear to shaft. In the design of machine mechanisms, it is necessary to know the magnitudes as well as the directions of forces transmitted from the input to the output. The analysis helps in selecting proper sizes of the machine components to withstand the stresses developed in them. If proper sizes are not selected, the components may fail during the machine operations. On the other hand, if the members are designed to have more strength than required, the machine may not be able to compete with others due to more cost, weight, size, etc.

If the components of a machine accelerate, inertia forces are produced due to their masses. However, if the magnitudes of these forces are small compared to the externally applied loads, they can be neglected while analysing the mechanism. Such an analysis is known as *static-force analysis*. For example, in lifting cranes, the bucket load and the static weight loads may be quite high relative to any dynamic loads due to accelerating masses, and thus static-force analysis is justified.

When the inertia effect due to the mass of the components is also considered, it is called *dynamic-force analysis* which will be dealt in the next chapter.

12.1 CONSTRAINT AND APPLIED FORCES

A pair of action and reaction forces which constrain two connected bodies to behave in a particular manner depending upon the nature of connection are known as *constraint forces* whereas forces acting from outside on a system of bodies are called *applied forces*.

Constraint forces As the constraint forces at a mechanical contact occur in pairs, they have no net force effect on the system of bodies. However, for an individual body isolated from the system, only one of each pair of constraint forces has to be considered.

Applied forces Usually, these forces are applied through direct physical or mechanical contact. However, forces like electric, magnetic and gravitational are applied without actual physical contact.

12.2 STATIC EQUILIBRIUM

A body is in static equilibrium if it remains in its state of rest or motion. If the body is at rest, it tends to remain at rest and if in motion, it tends to keep the motion. In static equilibrium

- the vector sum of all the forces acting on the body is zero, and
- the vector sum of all the moments about any arbitrary point is zero.